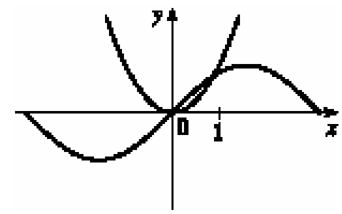
3. Since $x_1 = 3$ and $y = 5x - 4$ is tangent to $y = f(x)$ at $x = 3$, we simply need to find where the tangent line intersects the x-axis. $y = 0 \quad \Rightarrow \quad 5x_2 - 4 = 0 \quad \Rightarrow \quad x_2 = \frac{4}{5}$.

5. $f(x) = x^3 + 2x - 4 \quad \Rightarrow \quad f'(x) = 3x^2 + 2$, so $x_{n+1} = x_n - \dfrac{x_n^3 + 2x_n - 4}{3x_n^2 + 2}$.

Now $x_1 = 1 \quad \Rightarrow \quad x_2 = 1 - \dfrac{1 + 2 - 4}{3 \cdot 1^2 + 2} = 1 - \dfrac{-1}{5} = 1.2 \quad \Rightarrow \quad x_3 = 1.2 - \dfrac{(1.2)^3 + 2(1.2) - 4}{3(1.2)^2 + 2} \approx 1.1797$.

11. $\sin x = x^2$, so $f(x) = \sin x - x^2 \quad \Rightarrow \quad f'(x) = \cos x - 2x \quad \Rightarrow$



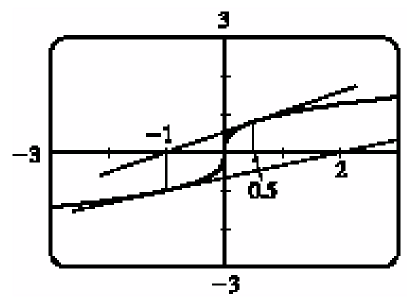
$x_{n+1} = x_n - \dfrac{\sin x_n - x_n^2}{\cos x_n - 2x_n}$. From the figure, the positive root of $\sin x = x^2$ is near 1. $x_1 = 1 \quad \Rightarrow \quad x_2 \approx 0.891396$, $x_3 \approx 0.876985$, $x_4 \approx 0.876726 \approx x_5$. So the positive root is 0.876726, to six decimal places.

21. (a) $f(x) = x^2 - a \quad \Rightarrow \quad f'(x) = 2x$, so Newton's method gives

$$x_{n+1} = x_n - \frac{x_n^2 - a}{2x_n} = x_n - \frac{1}{2}x_n + \frac{a}{2x_n} = \frac{1}{2}x_n + \frac{a}{2x_n} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right).$$

(b) Using (a) with $a = 1000$ and $x_1 = \sqrt{900} = 30$, we get $x_2 \approx 31.666667$, $x_3 \approx 31.622807$, and $x_4 \approx 31.622777 \approx x_5$. So $\sqrt{1000} \approx 31.622777$.

25. For $f(x) = x^{1/3}$, $f'(x) = \frac{1}{3}x^{-2/3}$ and



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^{1/3}}{\frac{1}{3}x_n^{-2/3}} = x_n - 3x_n = -2x_n.$$

Therefore, each successive approximation becomes twice as large as the previous one in absolute value, so the sequence of approximations fails to converge to the root, which is 0. In the figure, we have $x_1 = 0.5$, $x_2 = -2(0.5) = -1$, and $x_3 = -2(-1) = 2$.