Practice Problems for Midterm Exam #2

October 26th, 2007

1. Find the domain and the inverse of the following functions:
   (a) $f(x) = \ln (x-7)^3$,
   (b) $g(x) = \frac{3x-1}{2x+1}$
2. Find $(f^{-1})'(a)$.
   (a) $f(x) = x^4 + 3\sin x + 1$, $a = 1$
   (b) $f(x) = x\ln x + 1$, $a = 1$
3. Differentiate the functions:
   (a) $f(x) = (x+1)^{\ln x}$.
   (b) $f(x) = (\cos x)^{\frac{1}{x^2}}$.
   (c) $f(x) = (x + \sin x)^{\sin x}$.
   (d) $f(x) = \ln\left(\sin^{-1}(x+1)\right)$.
   (e) $f(x) = \cos^{-1}(\ln x)$.
   (f) $f(x) = \arctan(e^x)$.
4. Simplify:
   (a) $\sec(\arctan(\ln x))$.
   (b) $\tan(\arcsin(x))$.
5. Compute the following limits
   (a) $\lim\limits_{x\to\infty} \frac{4^x+x+1}{4^x+1}$
   (b) $\lim\limits_{x\to\infty} \left(\frac{1}{x} - \frac{1}{\ln x}\right)$
   (c) $\lim\limits_{x\to 0^+} x^5 \ln x$
   (d) $\lim\limits_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$
   (e) $\lim\limits_{x\to 0^+} (\tan x)^{\frac{1}{\ln \sin x}}$
   (f) $\lim\limits_{x\to 0^+} x\ln(\sin x)$

6. Find a such that

   (a) $\lim\limits_{x\to\infty}\left(1+\frac{a}{x}\right)^{\frac{x}{2}} = e^2$

   (b) $\lim\limits_{x\to 0}\frac{e^{ax}-2x-1}{x^2} = 2$

7. The population of the United States was 150 million on 1950 and 179 million on 1960. Assume that the growth rate is proportional to the population size. What is the relative growth rate? Use the model to predict the population in the year 2100.

8. Find the absolute maximum and absolute minimum of the following functions

   (a) $f(x) = x - \ln x^2$, on $[\frac{1}{\sqrt{e}}, \sqrt{e}]$

   (b) $f(x) = x^2(1-x)^3$, on $[0,1]$.

9. Prove that the function $f(x) = x^5 + x^3 + 1$ has neither an absolute maximum nor absolute minimum.

10. Find a such that the function $f(x) = \frac{ax}{x^2+1}$ has an absolute maximum at $x = -1$.

11. Suppose that $f(1) = 1$,$f(2) = 2$,$f(3) = 3$, $f(4) = 5$. Show that there exists c in $[1,5]$ such that $f''(c) \leq 1$.

12. Show that the equation $e^{-x} + x - 1 = 0$ has exactly one root in the interval $[-2,2]$

13. Show that the equation $3x + \cos x - 12 = 0$ has only one root.

14. Prove the following inequalities

    (a) $\ln x \leq x - 1$, for all $x \geq 1$.

    (b) $e^x \geq x + 1$, for all $x \geq 0$.

15. Find the intervals of increase or decrease, the interval of concavity, the local maximum and minimum values and the inflection points

    (a) $f(x) = x\sqrt{2-x^2}$

    (b) $f(x) = x^2 \ln x$

    (c) $f(x) = x^4 + \frac{8}{3}x^3 + a$

    (d) $f(x) = \frac{e^x}{x}$

    (e) $f(x) = \sqrt{\frac{x}{x+1}}$

16. Find the point of the hyperbola $y = \frac{1}{x}$ that is closest to the point $(0, 0)$.

17. Find the dimensions of a rectangle inscribed in a circle of radius 1 with the largest possible area.

18. Find the length of the longest rod which can be carried horizontally around a corner (90°) from a corridor 8 feet wide into one 4 feet wide.

19. A rectangle is to have an area of $64\ \mathrm{m}^2$. Find its dimensions so that the distance from one corner to the midpoint of a non-adjacent side shall be a minimum.

20. Apply Newton's method four times to approximate a root of the equation

    (a) $x^3 = 10$, starting with $x_1 = 3$.

    (b) $e^{-2x} - x - 2 = 0$, starting with $x_1 = -1$

21. Find f if $f''(x) = x^2 + \sin x + \cos x$, $f'(0) = 0$, $f(0) = 0$.

22. A particle starts to move with acceleration $a(t) = (e^t + t - 1)\mathrm{m/sec}$. How much distance will cover after 4 sec?