Practice Midterm Exam #1 (A)

October 2nd, 2007

Instructions: Show all of your work and write clearly and neatly. Calculators are NOT allowed on this test.

1. Consider the following function:

$$f(x) = \begin{cases} x, & x > 0 \\ 2, & x = 0 \\ x^2, & x < 0 \end{cases}$$

   (a) Sketch and label a graph of the function f.
   (b) Compute: $\lim_{x\to 3} f(x)$, $\lim_{x\to -3} f(x)$
   (c) Compute: $\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 0} f(x)$
   (d) List all vertical and horizontal asymptotes (if any).
   (e) Give one reason why this function is not continuous at $x = 0$. Is it possible to redefine f at $x = 0$ to make it continuous? Explain.

2. Compute the following limits
   (a) $\lim_{x\to\infty} \frac{x^2+1}{3x^2+x+2+\sin(x)}$
   (b) $\lim_{x\to 0} \frac{\sin(3x)}{\sin(x)+x}$
   (c) $\lim_{x\to\infty} x\left(\sqrt{x^2+2} - \sqrt{x^2+1}\right)$

3. Let $f(x) = 2x^3 + 1$. Use the definition of the derivative to compute $f'(x)$.

4. Let $f(x) = (1 + 2x + x^2)^5$.
   (a) Compute the first derivative.
   (b) Find the tangent line at $x = -1$.
   (c) Use the information from part (b) to compute an approximation for $f(-0.01)$.
   (d) Compute the second derivative.

5. Compute $\frac{dy}{dx}$ for y defined by $y(x) = \sin^3\left(\frac{\cos(x^2)}{\sqrt{5x}}\right)$.

6. Let $y(x)$be given implicitly by $\sin(\pi\sqrt{x/y}) + xy = 1$. Compute $\frac{dy}{dx}$at point (1,1)

7. Show that the equation $x^{10} + \sin(x) - 3 = 0$ has at least one root on $[1, 10]$.

8. A street light is mounted on top of a 15 foot tall pole. A man 6 feet tall walks away from the pole with a speed of 5 feet / sec along a straight path. How fast is the tip of his shadow moving when he is 40 feet from the pole? Hint: draw a diagram first.