Practice Midterm Exam #1 (B)

October 2nd, 2007

Instructions: Show all of your work and write clearly and neatly. Calculators are NOT allowed on this test.

1. Consider the following function:

$$f(x) = \begin{cases} \frac{x^2}{(x-1)(x-3)}, & x \neq 1 \\ 1, & x = 1 \\ 3, & x = 3 \end{cases}$$

   (a) Sketch and label a graph of the function f.
   (b) Compute: $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1+} f(x)$, $\lim_{x \to 1} f(x)$
   (c) List all vertical and horizontal asymptotes (if any).
   (d) Give one reason why this function is not continuous. Is it possible to redefine f at $x = 1$ and $x = 3$ to make it continuous? Explain.

2. Compute the following limits
   (a) $\lim_{x \to \infty} \frac{x^2+1}{x(1-2x)}$
   (b) $\lim_{x \to 0} \frac{x \tan(3x)}{\sin^2(2x)}$
   (c) $\lim_{x \to \infty} x\left(\sqrt{4x^2-1} - 2x\right)$

3. Let $f(x) = \frac{1}{x} + x$. Use the definition of the derivative to compute $f'(x)$.

4. Let $f(x) = \sin^3(1 - x/2)$.
   (a) Compute the first derivative.
   (b) Find the tangent line at $x = 2$.
   (c) Use the information from part (b) to compute an approximation for $f(1.99)$.
   (d) Compute the second derivative.

5. Compute $\frac{dy}{dx}$ for y defined by $y(x) = \sqrt{\frac{\sin(x)}{x} + (3x)^2}$.

6. Let $y(x)$be given implicitly by $y^2x + x^2y + \frac{x}{y} = 3$. Compute $\frac{dy}{dx}$at point (1,1)

7. Show that the equation $x + \sin(x^{10}) - 3 = 0$ has at least one root on $[1, 10]$.

8. The radius of a sphere is increasing at a rate of 3 cm / second. At what rate is the area of the surface of this sphere increasing when the radius is 25 cm? Hint: $S = 4\pi r^2$.