# Numerical Methods I
## MATH-GA 2010.001/CSCI-GA 2420.001

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Fall 2017, Thursday, 5:10–7:00PM, WWH #101

Sep. 7, 2017

# Outline

Organization issues

Introduction and examples

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- Email: If you email me (stadler@cims.nyu.edu) about anything related to this course, please put [num1] in the email's subject line.

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There is a part II of this class...

- ...in spring 2017 (taught by Jonathan Goodman). You should take both parts to get a reasonably complete overview of Numerical Methods.
- If you consider taking only one semester of Numerical Methods, I recommend taking Scientific Computing (taught by Leslie Greengard) instead of this class (same time, different place).

# Topics covered in Numerical Methods I

## Numerical Methods and their Analysis

- Stability; sources of errors; error propagation, representation of numbers in computers
- Numerical linear algebra: direct solution of sparse/dense linear system; solution of least square systems; eigenvalue problems; iterative solution of linear systems
- Nonlinear systems; Newton's method; Nonlinear least squares
- Numerical optimization
- Interpolation and Approximation
- Numerical integration

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## Computing Issues

- What makes some computer codes faster than others?
- Where are numerical methods used, and what is their role in science research?
- How large/complicated problems can we solve today? Where are the challenges and limits of what we can do?

# Topics of Numerical Methods II

Main topics covered in Spring 2018

- Approximation of ordinary differential equations (ODEs)
- Approximation of partial differential equations (PDEs)
- Solvers for the resulting (high-dimensional) discrete problems

# Programming

Programming the methods we discuss is an integral part of this course. To really understand methods & algorithms, one needs to implement them and experiment with them.

- Make sure you have access to MATLAB (CIMS, student license), you will need it for the first homework assignment.
- Alternatives to MATLAB: Octave, Python or Julia.
- We will talk about a few best coding practices, and how to present results.

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PDFs of Springer books can be downloaded for free (and legally)

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- An in-class final (40-50% of grade).

# Summary of resources:

- Books and homework assignments. I'll also make slides I use in class available.
- Piazza–central communication/discussion/announement platform
- Smile! These lectures are being video recorded; videos will be available through NYU Classes→Mediasite.
- Public class website: http://cims.nyu.edu/~stadler/num1/

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# Numerical mathematics

Computer simulations have had a big influence on research and development; sometimes the ability to simulate phenomena is referred to as the third pillar or science.

Numerical mathematics is a part of mathematics that develops, analyzes and applies methods from scientific computing to

- analysis
- linear algebra
- optimization
- differential equations
- ...

It has applications accross many applied sciences, including:

- physics
- economics
- biology
- finance
- ...

# Development of Numerical Methods at Courant

A few examples...

- Fast multipole method (FMM) (Greengard, O'Neil, Zorin, Cerfon)
- Immersed boundary method for solid-fluid interactions (Peskin)
- Adaptive mesh and cut cell methods for hyperbolic equations (Berger)
- Methods for studying dynamical systems, multiscale methods (Vanden-Eijnden)
- Methods for free boundary problems in fluid dynamics (Shelley)
- Scalable implicit solvers for viscous flows (Donev, Stadler)
- Sampling methods and Uncertainty Quantification (Goodman, Stadler)
- ...

# Applications of Numerical Methods at Courant

A few examples. . .

- Numerical simulation of Tsunami waves and flooding (Berger)
- Simulation and analysis of natural and artificial heart valves (Peskin)
- Simulation of plate tectonics and mantle convection (Stadler)
- The physics of cell's interiors and their motion (Shelley, Mogilner)
- Comutational fluid/hydrodynamics (Donev)
- Optimal complexity wave simulations (Greengard)
- Simulation of blood cells-resolving blood flow (Zorin)
- Plasmas and Magnetohydrodynamics (Cerfon)
- . . .

# Famous numerical mathematics failures

## Patriot Missile Failure

In the 1991 Golf War, a patriot missle failed to intercept an Iraqi Scud missile.

28 US soulders died, 100 were injured.

Cause: Inaccurate calculation of the time since boot due to computer arithmetic errors

http://www.ima.umn.edu/~arnold/disasters/patriot.html

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## Sinking of Sleipner oil platform

An oil platform in the North Sea sank near Stavanger (Norway) in 1991. Top part weights 57,000 tons, supposed to support drilling equipment that weights 40,000 tons.

Total economic loss was about 700 million USD.

Cause: Weak parts in the base could not resist the weight. Stresses were underestimated by 47%, leading to insufficient design. This was mainly due to an inaccurate finite element calculation to solve the PDE.

http://www.ima.umn.edu/~arnold/disasters/sleipner.html

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## Explosion of Ariane 5

Unmanned Ariane 5 rocket launched by the European space agency expolded in 1996.

Rocket value was abot 500 million USD.

Cause: Conversion of a floating point number to an integer led to “overflow” resulting in complete loss of guidance and altitude information 37 seconds after start.

http://www.ima.umn.edu/~arnold/disasters/ariane.html