Elliptic optimal control problems with L^1-control cost and applications for the placement of control devices *

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Correction of proof for Theorem 4.3.

Theorem 4.3 Let the initialization u^0 be sufficiently close to the solution $\bar{u}$ of P. Then the iterates u^k of Algorithm 1 converge superlinearly to $\bar{u}$ in $L^2(\Omega)$. Moreover, the corresponding states y^k converge superlinearly to $\bar{y}$ in $H_0^1(\Omega)$.

Proof To apply Theorem 4.1, it remains to show that the generalized derivative (4.7) is invertible and that the norms of the inverse linear mappings are bounded. Define $\mathcal{I} := \mathcal{I}_- \cup \mathcal{I}_+$, and for $\mathcal{S} \subset \Omega$ and $v \in L^2(\Omega)$ the restriction operator $E_\mathcal{S} : L^2(\Omega) \to L^2(\mathcal{S})$ by $E_\mathcal{S}(v) := v_{|\mathcal{S}}$. The corresponding adjoint opeator is the extension-by-zero operator $E_\mathcal{S}^* : L^2(\mathcal{S}) \to L^2(\Omega)$. To show that $\mathcal{G}(u)$ has a bounded inverse, we assume for arbitrary $w \in L^2(\Omega)$ that $\mathcal{G}(u)(v) = w$. From the explicit form (4.7), one immediately obtains that $E_{\Omega\setminus\mathcal{I}}$v = $E_{\Omega\setminus\mathcal{I}}$w. Thus, $v_\mathcal{I} := E_\mathcal{I} v \in L^2(\mathcal{I})$ satisfies

$$\alpha^{-1} E_\mathcal{I} A^{-\star} A^{-1} E_\mathcal{I}^\star v_\mathcal{I} + v_\mathcal{I} = E_\mathcal{I} w - \alpha^{-1} E_\mathcal{I} A^{-\star} A^{-1} E_{\Omega\setminus\mathcal{I}}^\star E_{\Omega\setminus\mathcal{I}} w. \qquad (*)$$

We now define the new scalar product $\langle \cdot\,, \cdot \rangle$ on $\mathcal{I}$ by

$$\langle v_1, v_2 \rangle := (v_1, v_2)_J + \alpha^{-1}(A^{-1}E_\mathcal{I}^\star v_1, A^{-1}E_\mathcal{I}^\star v_2)_\Omega,$$

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for $v_1, v_2 \in L^2(\mathcal{I})$. Clearly, $\langle \cdot\,, \cdot \rangle$ satisfies

$$\langle v_1, v_1 \rangle \geq (v_1, v_1)_{\mathcal{I}},$$

that is, the product $\langle \cdot, \cdot \rangle$ is coercive with constant 1 independently from $\mathcal{I}$. Using the Lax-Milgram lemma, one finds that $(*)$ admits a unique solution $v_{\mathcal{I}} \in L^2(\mathcal{I})$. Moreover, this solution satisfies

$$\|v_{\mathcal{I}}\|_{L^2(\mathcal{I})} \leq C\|w\|_{L^2(\Omega)}$$

with a constant $C > 0$ independent from $\mathcal{I}$ and thus from u. This proves the boundedness of $\mathcal{G}(u)^{-1}$ for all $u \in L^2(\Omega)$, which ends the proof. $\square$

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