Pellet Driven Disruptions in Tokamaks

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Abstract

Pellet injection can trigger ballooning like instability in tokamaks, driven by the large pressure perturbation of the pellet cloud. The instability is easier to excite when pellets are injected on the low magnetic field side of the tokamak, which is the usual case. This is because the pellet pressure perturbation tends to add to the background pressure drive. When pellets are injected on the high field side, the instability is harder to excite, because the pellet pressure perturbation opposes the the background pressure drive. Nonlinearly, the instability causes fragmentation of the pellet cloud, and can give anomalous pellet penetration. This effect might explain recent experimental observations.

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I. Introduction

Pellet injection can trigger ballooning like instabilities in tokamaks, driven by the large pressure perturbation of the pellet. The instability is easier to excite when pellets are injected on the low magnetic field side of the tokamak, which is the usual case. This is because the pellet pressure perturbation tends to add to the background pressure drive. When pellets are injected on the high field side, the instability is harder to excite, because the pellet pressure perturbation opposes the the background pressure drive. Nonlinearly, the instability causes fragmentation of the pellet cloud, and can give anomalous pellet penetration. This effect might help explain recent experimental observations [1] of anomalous penetration of "killer pellets" in the Doublet III D (DIIID) experiment.

Earlier nonlinear magnetohydrodynamic (MHD) simulations of pellet injection [2] are in qualitative agreement with recent Axisymmetric Divertor Experiment (ASDEX) results [3]. MHD forces accelerate pellets in the direction of increasing major radius. Pellets injected on the high field side of a tokamak, if large enough, can penetrate to the plasma center. These simulations were MHD stable, because the initial equilibrium pressure gradient was well below the instability threshold, and the additional pressure perturbation of the pellet was insufficient to cause instability. In the present work, we start with initial equilibria which are closer to being unstable, and the additional pressure perturbation of the pellet can produce instability. The effect is similar to pressure driven disruptions triggered by internal kink modes [4], which locally increase the pressure gradient above the instability threshold.

The threshold depends on the location of the pellet cloud, for a given pressure perturbation. Inboard pellet injection confers advantages with regards to MHD stability of the background plus pellet system. Because of the high local β at the pellet cloud, the system may be unstable to pressure driven modes, even though the background equilibrium is stable. Pellets on the outboard side tend to be more destabilizing, because the pellet pressure gradient and the equilibrium gradient add on the large major radius side of the equilibrium. With inboard injection, the pressure gradients oppose one another. In addition, on the low field side, the velocity perturbations resemble typical moderate wavelength ballooning modes. They produce disruptions in nonlinear simulations. On the high field side, the velocity perturbations are much more localized.

In Section II, we review the physical model, including our pellet model. Section

III describes the computational approach. Section IV discusses numerical examples of pressure driven instabilities caused by the presence of a pellet cloud. Section V provides a ballooning theory of the instability, which is in agreement with the simulations. Section VI contains nonlinear simulations of pellet pressure driven instabilities, showing fragmentation and anomalous penetration of the pellet cloud. Conclusions are given in Section VII.

II. Physical Model

When a pellet is injected into a plasma, the pellet rapidly heats, and ablates. The ablated gas then ionizes and becomes a cloud of high density plasma, which is cooler than its surroundings. The plasma cloud moves together with the background plasma, according to the dissipative MHD equations,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \tag{1}$$

$$\rho \frac{d\mathbf{v}}{dt} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{v} \tag{2}$$

$$p = \rho T \tag{3}$$

where η is the resistivity and μ a scalar viscosity. The density satisfies the continuity equation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \mathbf{v} \tag{4}$$

A single temperature is assumed for simplicity, whose evolution equation includes a large thermal transport parallel to the magnetic field, to be described below in (9) and (10). This tends to make the temperature approximately constant on magnetic field lines.

An initial non equilibrium state, will move towards equilibrium, because it has lower potential energy. An MHD equilibrium satisfies

$$\nabla_{\|} p = 0. \tag{5}$$

The initial conditions for the simulations are obtained assuming that the ablation, ionization and parallel thermal conduction time scales are short compared to the typical Alfvén time for MHD motion. The details of the ablation process [5] are not considered. This is done to separate the purely MHD effects on pellet cloud displacement from the details of the ablation process, and to reduce parameter space.

When pellet material ablates, it cannot be distributed uniformly on magnetic flux surfaces, because the non ionized material is not constrained by the magnetic field.

We assume the injection process is adiabatic, imparting no energy to the plasma. The temperature is constant on flux surfaces before and after the pellet injection. After injection, the temperature is lower on flux surfaces that intersect the pellet cloud. To calculate the new temperature, adiabaticity implies that the pressure contained between flux surfaces is the same, before and after the pellet injection. Equivalently, the flux surface average $< p >$ of the pressure p remains the same, where the flux surface average is

$$< p >= \frac{\int d\phi d\ell p/B}{2\pi \int d\ell/B} \tag{6}$$

and ϕ is the toroidal angle. Let p_0, ρ_0, and T_0 refer to the unperturbed state before the pellet, and $p_0 + \delta p, \rho_0 + \delta\rho$, and $T_0 + \delta T$ refer to the perturbed state containing the pellet. The temperature is a flux function, $T_0 =< T_0 >$, $\delta T =< \delta T >$, as well as the initial density, $\rho_0 =< \rho_0 >$. Adiabaticity $< \delta p >= 0$ implies that

$$\delta T = -\frac{< \delta\rho >}{\rho_0 + < \delta\rho >} T_0, \tag{7}$$

which shows how the temperature is reduced by the density perturbation, such that T is a flux function. The pellet - perturbed density $\delta\rho$ is not constant on a flux surface. The ratio of peak to average density is large, and the density ablation cloud is localized in three dimensions, so (5) is far from being satisfied. The pellet perturbed pressure is given by

$$\delta p = \frac{\delta\rho - < \delta\rho >}{\rho_0 + < \delta\rho >} p_0. \tag{8}$$

The flux surface averaged pressure (8) is unchanged by the pellet cloud, but locally the pressure varies on flux surfaces. This can lead to MHD instability, because the unstable modes do not sample the pressure distribution on flux surfaces uniformly.

The adiabatic model is suitable for hydrogen pellets, but is not necessarily valid for "killer" impurity pellets, which radiate a large amount of energy. The lack of adiabaticity may tend to reduce the pressure perturbation of killer pellets, so that they may be more MHD stable. A better assessment of the stability of plasmas containing killer pellets will require realistic ablation modeling [6].

The density blob spreads out because of plasma compression effects. The MH3D code treats both fast and slow magnetosonic waves, which both contribute to the evolution of the density. The fast magnetosonic waves cause the relatively rapid expansion and equilibration of the pellet cloud pressure with the magnetic pressure. On a slow sound waves time scale, motion along parallel to $\mathbf{B}$ with $v_\parallel \neq 0$ causes the density to spread along magnetic field lines.

III. Computational Model

The MH3D++ code has been used to simulate the 3D dynamics of pellet injection. MH3D++ is a modification of the PPPL MH3D resistive full MHD code [7, 8], which replaces the original finite difference / spectral discretization, with an unstructured mesh and finite element [9] / spectral discretization. The unstructured mesh is made with triangular and quadrilateral cells.

The MHD equations are discretized with a zero residual Galerkin approach, using piecewise linear basis functions. The differential operators in the equations become sparse matrices involving integrals of the basis functions and their derivatives.

The finite element unstructured mesh discretization has been incorporated into MH3D++ with an object oriented approach. The unstructured mesh objects generate an unstructured mesh and produce the sparse matrices which implement differential operators including gradient, curl, and divergence, as well as various Poisson solvers based on an Incomplete Cholesky Conjugate Gradient solver. Also included are mesh operations needed for line integration and contour plots.

An important feature of this approach is that most of the MH3D code is retained. This allows direct benchmarking of the two versions against each other. Equilibrium and stability calculations using the two versions have been compared, and agree well.

To represent the effect of electron free streaming along magnetic field lines, or large parallel temperature transport, the MH3D code uses the "artificial sound" method [10]. In this approach, an artificial parallel velocity $v_{a\parallel}$ is introduced, satisfying

$$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla T - (\gamma - 1) T \nabla \cdot \mathbf{v} + \frac{B}{\rho} \nabla_\parallel v_{a\parallel} + \chi \nabla^2 T \tag{9}$$

$$\frac{\partial v_{a\parallel}}{\partial t} = -\frac{1}{\tau_a} v_{a\parallel} + c_a B \nabla_\parallel T \tag{10}$$

The constants c_a, τ_a are chosen so that the temperature relaxes rapidly to a near equilibrium [10]. The temperature diffusivity χ is small and improves numerical stability. In the absence of damping and dissipation, the system (9), (10) conserves the flux tube averaged pressure as well as an "artificial energy", so that in the presence of damping the system approaches an equilibrium. The system (9), (10) models electron free streaming along field lines, which is the cause of temperature equilibration.

The following computations were done on a poloidal mesh of slightly more than 3000 grid points, and at least 8 toroidal Fourier harmonics in addition to the $n = 0$ mode. In order to adequately represent a more toroidally localized pellet cloud, more toroidal harmonics would be required. The plasma is assumed bounded by a rigid conducting wall.

An initial equilibrium was prepared, starting with a prescribed initial, non equilibrium state, and evolving in 2D, removing kinetic energy, until an equilibrium is approached. In the following cases, the equilibrium has a rotational transform at the magnetic axis $q_0 = 1.1$, aspect ratio $R/a = 4$, and a peak $\beta = 0.15a/R$. In addition, the pressure is boosted an additional constant amount with $\beta = 0.15a/R$. This additional constant pressure enhances the sound speed, while having no effect on the background magnetic equilibrium. This makes the total peak $\beta = 0.30a/R$. The initial equilibrium density is constant. If the pressure gradient is doubled, the equilibrium is unstable.

The initial equilibrium was perturbed with a density blob representing the ionized pellet ablation cloud. In the following, the blob's peak density is up to 15 times the background density. The blob is cigar shaped, with a circular cross section in the poloidal plane, and a dependence on toroidal angle ϕ proportional to $\sim \cos^2(\phi/2)$. The density perturbation extends less than half way around the torus. The model ablation cloud is considerably less localized than in experiments. This was done because of constraints on numerical resolution, that should be relaxed in future simulations. Similarly the density contrast is less than in experiments. Higher resolution simulations will be required to model experiments more accurately.

The temperature was initialized adiabatically, as described in Section II. Before the pellet is injected, the temperature is given by $T = p_0(\psi)/\rho_0$, where ψ is the magnetic flux function. The flux surface averaged density (6) is used to calculate a new temperature. The new pressure is then obtained from (3). This gives an initial pellet state with a pressure highly nonuniform on flux surfaces, although the flux surface averaged pressure is unchanged. This causes a 3D motion towards a new equilibrium state. It can also cause faster, more spatially localized motions, which are very similar to pressure driven instability in axially symmetric equilibria.

IV. “Linear” Stability

A high β equilibrium was produced, as described in the previous section. This was modified by the presence of a pellet perturbation. Inboard and outboard pellet perturbations were centered on approximately the same flux surface, so that a given density perturbation produced the same amplitude pressure perturbation. The density perturbation is $\delta\rho/\rho \leq 15$.

Unlike a true stability calculation, the background state was not an equilibrium. Instead it consisted of an equilibrium with a pellet perturbation, which is not an equilibrium. The rationale is to look for instabilities that grow on a time scale short compared to the toroidal sound wave transit time, in which the pellet pressure perturbation smoothes out. These instabilities, as will be shown in nonlinear simulations, modify the pellet dynamics before it has a chance to equilibrate along the magnetic field. The linear simulations were performed by starting with an initial state of equilibrium and pellet, and adding a small 3D velocity perturbation. The system was then time advanced, with the velocity field being rescaled to keep the kinetic energy in each toroidal harmonic below some small preset level. This ensured that the motion remained “linear.” The growth rate of the kinetic energy in harmonics with toroidal mode number $n > 3$ was monitored to obtain a growth rate.

An outboard pellet is considered in Fig.1, Fig.2. The initial pressure profile as a function of major radius, $p(R)$, through the maximum of the pellet pressure perturbation, is shown in Fig.1. The density perturbation is $\delta\rho/\rho = 15$. The shape of the unstable modes is shown in Fig.2 showing contours of electrostatic potential. This mode has a mixture of toroidal mode numbers, centered at $n = 5$. The mode has a ballooning structure, and is toroidally localized, but poloidally less localized than the following inboard injection case. This is similar to nonlinear destabilization by n=1 pressure perturbations during disruptions [4].

In comparison, an inboard pellet is shown in Fig.3, Fig.4. The pressure profile is shown in Fig.3. The density perturbation is the same, $\delta\rho/\rho = 15$, and so is the pressure perturbation. The electrostatic potential contours of the unstable mode are shown in Fig.4. The mode is more localized to the pellet, because outside the pellet, the pressure gradient is in the same direction as the curvature, which is stabilizing. This difference appears to persist in nonlinear runs, using the pellet and linear modes as perturbations of the background equilibrium. In this case too, inboard injection is more favorable, because the instability threshold $\delta\rho/\rho$ is higher, and because the unstable modes are more localized.

Growth rates are summarized in Fig.5. The growth rate as plotted as a function of $\delta\rho/\rho$. The upper curve is for outboard pellet perturbations, while the lower curve is for inboard. For inboard pellet perturbations, the growth rate is lower, and the threshold $\delta\rho/\rho$ is higher. Hence, inboard injection is favorable not just from the standpoint of pellet penetration, but also with respect to pellet - induced disruptions. Inboard injection is more favorable, because the instability threshold $\delta\rho/\rho$ is higher, and because the unstable modes are more localized.

These results indicate that pellets can cause MHD instability, and that the stability threshold depends on the location of the pellet. The stability threshold also depends on details of the pellet cloud localization, including its toroidal and poloidal extent. The simulational results are in reasonable agreement with the following ballooning stabilitity theory.

V. Ballooning Theory

Stability to ballooning modes is given by the energy principle [11]

$$\delta W = \int [N^2 B^2 |\frac{d\phi}{d\ell}|^2 + 2\kappa \cdot \mathbf{N} p' |\phi|^2] d\ell \tag{11}$$

where $\mathbf{N} = (\nabla\chi) \times \mathbf{B}/B^2$, χ is the phase of the ballooning mode, such that $\mathbf{B} = \nabla\psi \times \nabla\chi$; $\kappa = (\mathbf{B}/B) \cdot \nabla(\mathbf{B}/B)$, and $p' = \partial p/\partial\psi$. The pressure has two parts,

$$p = p_0 + p_p$$

where p_0 is the background pressure and p_b is the pellet cloud pressure. We split off the contribution of the pellet to $\delta W = \delta W_0 + \delta W_p$. Let us assume the pellet toroidal length L_p is small compared to the toroidal circumference. We can then approximate

$$\delta W_p = 2\kappa(\mathbf{r}_p) \cdot \mathbf{N} p_p'(\mathbf{r}_p) |\phi(\mathbf{r}_p)|^2 L_p \tag{12}$$

where $\mathbf{r}_p$ is the location of the pellet.

We can now make a simple circular cross section zero shear approximation to evaluate the rest of δW. Let $d\ell = qRd\theta$, $\chi = q\theta - \phi$, $\mathbf{N} = q\nabla r/rB$, $\kappa = (2/R)\cos(\theta)$. This gives

$$\delta W_p = -2\frac{q^2}{Rr^2B^2}\frac{p_b}{r_p}|\phi(\mathbf{r}_p)|^2 L_p,$$

where r_p is the pellet cloud radial gradient scale length. Let $\phi = \phi(\mathbf{r}_p)\cos^2(\theta/2)$ for a pellet on the outboard side, while $\phi = \phi(\mathbf{r}_p)\sin^2(\theta/2)$ for a pellet on the inboard side, so that

$$\delta W_0 = 2\pi\frac{q^3}{r^2B^2}\left(\frac{B^2}{8q^2R} \mp \frac{p_0}{2r_0}\right)|\phi(\mathbf{r}_p)|^2$$

where r_0 is the background pressure gradient scale length, the negative sign in the second term is for an outboard pellet, and the positive sign is for an inboard pellet. Combining, the total δW is

$$\delta W = 2\pi\frac{q^3R}{r^2B^2}\left(\frac{B^2}{8q^2R} \mp \frac{p_0}{2r_0} - \frac{\alpha_p p_b}{\pi q R}\right)|\phi(\mathbf{r}_p)|^2, \tag{13}$$

where we introduced $\alpha_p = L_p/r_p$, the pellet cloud aspect ratio.

The ratio of the last two terms in $\delta W$is of order unity. The pellet cloud β cannot be much larger than unity, or else the cloud will expand rapidly (on the magnetosonic time scale) until β is of order unity.

Remarkably, the size and mass of the pellet cloud do not enter the calculation directly. This is because the pellet cloud size must adjust self consistently in order to have the condition $\beta \leq 1$ in the cloud. The size and mass of the pellet cloud enter only in the calculation of p_b. Note that after the pellet has expanded radially, it expands along the magnetic field, such that the pellet cloud density is inversely proportional to its length, by conservation of mass. While the cloud expands along the magnetic field, the temperature remains constant on field lines, with the result that $\alpha_p p_b$ stays constant, equal to its value when the pellet cloud stops expanding radially. Hence $\alpha_p p_b$ in the last term is limited, approximately, to $\alpha_p p_b \leq B^2/2$.

In Fig.5 calculated numerically in the previous section, for an outboard pellet, the marginal pellet density is 2/15 of its value in Fig.1, or $p_b = 0.02a/R$, where a is the minor radius. Here the toroidal length of the pellet cloud is relatively large, $\alpha_p r_p = \pi R/2$, and $r_p = 0.2a$, $B = 1$, $r_0 = 2a$, $a = 1, R = 4$, $q = 1.5$, so that $\alpha_p p_b/\pi q R = 0.03/R$. The second term has $p_0/4a = 0.04/R$. The two destabilizing terms are roughly equal in magnitude for an outboard pellet. The first, stabilizing term, is $0.06/R$, so there is an approximate balance of stabilizing and destabilizing terms.

In the case of inboard injection, the middle and last terms are of about equal magnitude, but the middle term now has a positive sign, so the last term has to be about three times larger to balance the first term. This is only roughly consistent with Fig.5, which shows that the critical p_b for inboard injection is about 2 times larger than for outboard injection.

VI. Nonlinear Simulations

Nonlinear simulations of disruptions induced by pellet perturbations indicate anomalous penetration of the pellet material. The instability causes fragmentation of the larger major radius side of the pellet cloud, while the inner part is relatively less disturbed.

Nonlinear simulations were performed with similar initial states as the linear cases. Both inboard and outboard cases had $\delta\rho/\rho = 15$. The pellet positions are somewhat different from the linear cases. The nonlinear runs were initialized by starting with an equilibrium (the same as in the previous section), adding a pellet perturbation, as well as a small velocity perturbation. The qualitative result, that there is a net density pinch, is independent of the sign of the velocity perturbation.

In the case of outboard injection, the initial density is shown in Fig.6. At time $t = 7.76\tau_A$, shown in Fig.7, the density at first evolves without instability. The density contours spread poloidally as well as toroidally, along the magnetic field. Later, ripples appear in the density contours. The density peak is pushed inward in the central midplane, and outwards above and below the midplane. At time $t = 40.36\tau_A$, shown in Fig.8, the density perturbations become more pronounced, as well as more extended poloidally.

The motion of the density distribution can also be seen by considering the flux surface averaged density, performing the flux surface average in the toroidally averaged magnetic field. This technique was used in a previous study to show the tendency of a pellet density perturbation to move outward to larger major radius [2]. The flux averaged density, as a function of averaged poloidal flux ψ, is shown in Fig.9, for the outboard case. The initial flux surface averaged density $< \rho > (\psi)$ is labeled (a). The density at time $t = 7.76\tau_A$ is shown in curve (b). As expected, the peak of the curve has shifted to the right, corresponding to a shift towards the outer boundary at $\psi = 0$. The density at $t = 40.36\tau_A$ is shown in curve (c). Now the peak of the density has shifted inwards, to the left of the initial density peak. The density has shifted inwards towards the magnetic axis.

A plausible cause for this inward transport is the change in the density distribution as the density perturbation flows along the magnetic field. An isoplot in Fig.10

shows the initial density and Fig.11 shows the density at $t = 7.76\tau_A$. At this point significant density perturbations appear halfway around the torus. Later, as the density continues to spread along the magnetic field, there is a substantial density perturbation on the inside of the torus. This becomes more pronounced in Fig.12 at $t = 40.36\tau_A$. In fact it appears that the density perturbation is larger on the inside of the torus than on the outside. This can be attributed to the mixing caused by the ballooning like modes, which are localized to the outside, bad curvature part of the torus. The density distribution now resembles a pellet perturbation centered on the *inside* of the torus. As such, it tends to move radially outward, carrying the pellet material towards the magnetic axis.

The case of inboard injection is somewhat similar. The initial density is shown in Fig.13. At time $t = 7.5\tau_A$, shown in Fig.14, the pellet spreads poloidally (and toroidally). The instability causes a slight splitting of the density peak. Later, the pellet density becomes rippled, on the side closest to the plasma center. This is where the ballooning instability perturbations are strongest, as seen in Fig.4. At time $t = 41.1\tau_A$, shown in Fig.15, the perturbations become large and spread across the pellet contours.

The flux averaged density for the inboard case is shown in Fig.16. The initial density $< \rho > (\psi)$ is shown in curve (a). The flux averaged density at $t = 41.1\tau_A$ is shown in curve (b). The density peak is shifted toward the magnetic axis. Here too the net effect is carry the density inwards towards the plasma center.

VII. Conclusion

Pellet injection causes large pressure perturbations on magnetic flux surfaces. The perturbations can drive MHD instability, when the pressure perturbations exceed a threshold. For a given pellet density centered on a particular magnetic surface, the threshold for instability is higher if the pellet is on the inboard side, rather than the outboard side. This is reasonable intuitively, because where the inboard pellet pressure profile is ballooning unstable, the equilibrium pressure gradient is stable. In the outboard case, the pellet and background pressure gradients add on the outside edge of the plasma, enhancing the ballooning instability.

The simulations modeled pellet clouds with much less spatial localization than in experiments, and should be regarded as qualitative rather than quantitative.

Nonlinearly, the instability causes distortion and ultimately fragmentation of the pellet density cloud. The instability causes the peak of the density perturbation to move inwards towards the magnetic axis, causing anomalous penetration of the pellet. For outboard pellet injection, this opposes the tendency for the pellet cloud to be displaced outwards in major radius. This effect may help explain the anomalous pellet penetration seen in “killer pellet” disruption experiments.

Acknowledgments

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Figure Captions

Fig.1 Pressure as a function of major radius through the center of an outboard pellet.

Fig.2 Contours of electrostatic potential for outboard pellet.

Fig.3 Pressure as a function of major radius through the center of an inboard pellet.

Fig.4 Contours of electrostatic potential for inboard pellet.

Fig.5 Growth rate as a function density perturbation.

Fig.6 Density at $\phi = 0, t = 0$.

Fig.7 Density at $\phi = 0, t = 7.76\tau_A$

Fig.8 Density at $\phi = 0, t = 40.36\tau_A$

Fig.9 Flux averaged density at (a) $t = 0$, (b)$t = 7.76\tau_A$, (c)$t = 40.36\tau_A$.

Fig.10 Iso plot of density at $t = 0$.

Fig.11 Iso plot of density at $t = 7.76\tau_A$

Fig.12 Iso plot of density at $t = 40.36\tau_A$

Fig.13 Density at $\phi = 0, t = 0$.

Fig.14 Density at $\phi = 0, t = 7.5\tau_A$

Fig.15 Density at $t = 41.1\tau_A$

Fig.16 Density at (a) $t = 0$, (b)$t = 32.4\tau_A$, (c)$t = 41.1\tau_A$.