# MHD Effects on Pellet Injection in Tokamaks

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**Abstract**

The location at which pellets are injected into a plasma can have a significant effect on what fraction of the pellet mass remains in the plasma for refueling purposes. MHD simulations presented here, confirm the results of pellet injection experiments: toroidal curvature makes it favorable to inject pellets from the inboard side or from the top or bottom, rather than from the outboard side. Pellets injected at the inboard edge can reach the plasma center, and in the process drive magnetic reconnection to produce negative central shear. Injection at the top (or bottom) of the tokamak causes relatively little MHD displacement of the pellet. A scaling law is obtained for pellet displacement which agrees well with the simulations. The MHD simulations were carried out with the new MH3D++ [1] unstructured mesh finite element version of the MH3D [2] full MHD code.

## I. Introduction

Pellet injection is a primary method of replenishing density loss in large size, long pulse tokamaks, such as ITER, which are intended to operate for many particle confinement times [3]. The method has been tried in many experiments, and is generally considered successful, although a significant fraction of the pellet mass can be lost, not absorbed into the plasma.

The location at which pellets are injected has an effect on how much of the pellet mass is retained in the plasma. Usually, pellets are injected from the outboard, large major radius edge of the plasma, because this is the most accessible part of the experimental apparatus. Recent ASDEX experiments [4] demonstrated that pellets injected on the inboard, low major radius edge, suffered less loss, and were absorbed more completely into the plasma.

For very large tokamaks, such as ITER, there is a further requirement to get pellets to "fall into" the center using this mechanism, because they will ionize relatively near the plasma edge.

The cause of this effect is the toroidal curvature, the same effect responsible for the outward Shafranov shift of the flux surfaces, acting in conjunction with expansion of the ionized pellet ablation cloud. We demonstrate the effect using MHD simulations.

The numerical results confirm that pellets injected at the outboard edge lead to an MHD equilibrium in which the injected density moves relatively closer to the outboard plasma edge. We also show that sufficiently massive pellets, injected at the inboard side of the plasma, are able to reach the plasma center. In the process, the pellet drives magnetic reconnection leading to a reversed shear profile. This may offer a way of maintaining reversed negative central shear, which can be highly favorable for transport, in the presence of Ohmic dissipation in large long pulse or steady state tokamaks.

Finally, pellets injected at the top (or bottom) edge lead to an MHD equilibrium in which the injected density moves relatively little.

We derive a simple model of the pellet displacement, which agrees well with the numerical data obtained from the 3D simulations. The pellet displacement is proportional to the pellet pressure perturbation, and the cosine of the angle between the normal to the flux surface at the pellet and the major radius direction.

## II. Physical Model

When a pellet is injected into a plasma, the pellet rapidly heats, and ablates. The ablated gas then ionizes and becomes a cloud of high density plasma, which is cooler than its surroundings. The plasma cloud moves together with the background plasma, according to the dissipative MHD equations,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \tag{1}$$

$$\rho \frac{d\mathbf{v}}{dt} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{v} \tag{2}$$

$$p = \rho T \tag{3}$$

where $\eta$ is the resistivity and $\mu$ a scalar viscosity. The density satisfies the continuity equation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \mathbf{v} + D \nabla^2 \rho \tag{4}$$

where $D$ is a small diffusion coefficient, kept to improve numerical stability. A single temperature is assumed for simplicity, whose evolution equation includes a large thermal transport parallel to the magnetic field, to be described below in (12) and (13).

This tends to make the temperature approximately constant on magnetic field lines,

$$\nabla_{||} T = 0. \tag{5}$$

An initial non equilibrium state, will move towards equilibrium, because it has lower potential energy. An MHD equilibrium satisfies

$$\nabla_{||} p = 0. \tag{6}$$

which together with (5) requires

$$\nabla_{||} \rho = 0. \tag{7}$$

The initial conditions for the simulations are obtained assuming that the ablation, ionization and parallel thermal conduction time scales are short compared to the typical Alfvén time for MHD motion. The details of the ablation process [5] are not considered, but may be taken into account in in future simulations. The most important fact is that when the pellet material ionizes, it cannot be distributed uniformly on magnetic flux surfaces, because the non ionized material is not constrained by the magnetic field.

In principle, the injection process is adiabatic, imparting no energy to the plasma. The temperature is constant on flux surfaces before and after the pellet injection. After injection, the temperature is lower on flux surfaces that intersect the pellet cloud. To calculate the new temperature, adiabaticity implies that the pressure contained between flux surfaces is the same, before and after the pellet injection. Equivalently, the flux surface average $< p >$ of the pressure $p$ remains the same, where the flux surface average is

$$< p > = \frac{\int d\phi d\ell p/B}{2\pi \int d\ell/B} \tag{8}$$

and $\phi$ is the toroidal angle. Let $p_0, \rho_0$, and $T_0$ refer to the unperturbed state before the pellet, and $p_0 + \delta p, \rho_0 + \delta\rho$, and $T_0 + \delta T$ refer to the perturbed state containing the pellet. The temperature is a flux function, $T_0 =< T_0 >$, $\delta T =< \delta T >$, as well as the initial density, $\rho_0 =< \rho_0 >$. Adiabaticity $< \delta p >= 0$ implies that

$$\delta T = -\frac{< \delta\rho >}{\rho_0 + < \delta\rho >} T_0, \tag{9}$$

which shows how the temperature is reduced by the density perturbation, such that $T$ is a flux function. The pellet - perturbed density $\delta\rho$ is not constant on a flux surface.

The ratio of peak to average density is large, and the density ablation cloud is localized in three dimensions, so (7) is far from being satisfied. The pellet perturbed pressure is given by

$$\delta p = \frac{\delta\rho - <\delta\rho>}{\rho_0 + <\delta\rho>} p_0. \tag{10}$$

The flux surface averaged pressure perturbation (10) vanishes, but the unaveraged perturbed pressure is not constant on magnetic field lines. Hence (6) is not satisfied, and the plasma is out of MHD equilibrium. The subsequent motion of the plasma will be studied in numerical simulations.

The density blob spreads out because of plasma compression effects. Eqs. (1) and (4) lead to (neglecting diffusion)

$$(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla)(\mathbf{B}\cdot\nabla\ln\rho) = \frac{1}{\rho}\mathbf{B}\cdot\nabla(\rho\nabla\cdot\mathbf{v}) \tag{11}$$

If $\nabla\cdot\mathbf{v} \neq 0$, the density gradient along the magnetic field can change. The MH3D code treats both fast and slow magnetosonic waves, which both contribute to the evolution of the density. The fast magnetosonic waves cause the relatively rapid expansion and equilibration of the pellet cloud pressure with the magnetic pressure. On a slow time scale, sound waves with $v_{||} \neq 0$ cause the density to equalize on magnetic field lines.

## III. Computational Model

The MH3D++ code has been used to simulate the 3D dynamics of pellet injection. MH3D++ [1] is a modification of the PPPL MH3D resistive full MHD code [2], which replaces the original finite difference / spectral discretization, with an unstructured mesh and finite element [6],[7] / spectral discretization. The unstructured mesh is made with triangular and quadrilateral cells. A sample mesh similar to that used in the computations, but with only 1/4 as many mesh points for clarity, is shown in Fig.1.

The MHD equations are discretized with a zero residual Galerkin approach, using piecewise linear basis functions. The differential operators in the equations become sparse matrices involving integrals of the basis functions and their derivatives.

The finite element unstructured mesh discretization has been incorporated into MH3D++ with an object oriented approach. The unstructured mesh objects generate an unstructured mesh and produce the sparse matrices which implement differential operators including gradient, curl, and divergence, as well as various Poisson solvers based on an Incomplete Cholesky Conjugate Gradient solver. Also included are mesh operations needed for line integration and contour plots.

An important feature of this approach is that most of the MH3D code is retained. This allows direct benchmarking of the two versions against each other. Equilibrium and stability calculations using the two versions have been compared, and agree well.

To represent the effect of electron free streaming along magnetic field lines, or large parallel temperature transport, the MH3D code uses the “artificial sound” method [8]. In this approach, an artificial parallel velocity $v_{a\|}$ is introduced, satisfying

$$\frac{\partial T}{\partial t} = -\mathbf{v}\cdot\nabla T - (\gamma - 1)T\nabla\cdot\mathbf{v} + \frac{B}{\rho}\nabla_{\|}v_{a\|} + \chi\nabla^2 T \tag{12}$$

$$\frac{\partial v_{a\|}}{\partial t} = -\frac{1}{\tau_a}v_{a\|} + c_a\rho B\nabla_{\|}T \tag{13}$$

The constants $c_a, \tau_a$ are chosen so that the temperature relaxes rapidly to a near equilibrium (5). The temperature diffusivity $\chi$ is small and improves numerical stability. In the absence of damping and dissipation, the system (12), (13) conserves the flux tube averaged pressure as well as an ”artificial energy”, so that in the presence of damping the system approaches an equilibrium (5). The system (12), (13) models electron free streamimg along field lines, which is the cause of temperature equilibration.

The following computations were done on a poloidal mesh of slightly more than 2000 grid points, and at least 9 toroidal Fourier harmonics in addition to the $n = 0$ mode. The plasma is assumed bounded by a rigid conducting wall.

An initial equilibrium was prepared, starting with a prescribed initial, non equilibrium state, and evolving in 2D, removing kinetic energy, until an equilibrium is approached. In the following three cases, the equilibrium has a rotational transform at the magnetic axis $q_0 = 1.7$, initial peak $\beta = 0.06a/R$, and the aspect ratio $R/a = 5$. In addition, the pressure is boosted an additional constant amount with $\beta = 0.3a/R$. This additional constant pressure enhances the sound speed, while having no effect on the background magnetic equilibrium. This makes the total peak $\beta = 0.36a/R$. The initial equilibrium density is constant. The equilibrium flux surfaces are shown in Fig.2, and the $q(\psi)$ profile in Fig.3.

The initial equilibrium was perturbed with a density blob representing the ionized pellet ablation cloud. In the following, the blob's peak density is up to 15 times the background density. The blob is cigar shaped, with a circular cross section in the poloidal plane, and a dependence on toroidal angle $\phi$ proportional to $\sim \cos^4(\phi/2)$. The density perturbation extends roughly 1/4 of the way around the torus. The model ablation cloud is considerably less localized than in experiments. This was done because of constraints on numerical resolution, that should be relaxed in future simulations. Similarly the density contrast is less than in experiments. Nonetheless, the effects demonstrated here are expected to be sufficiently robust to qualitatively account for experimental observations.

The temperature was initialized adiabatically, as described in Section II. Before the pellet is injected, the temperature is given by $T = p_0(\psi)/\rho_0$, where $\psi$ is the magnetic flux function. After the pellet is inserted, the density is flux surface averaged, as in (8), and used to calculate a new temperature, (9). The new pressure is then obtained from (3). This gives an initial pellet state with a pressure highly nonuniform on flux surfaces, although the flux surface averaged pressure is unchanged. This causes a 3D motion towards a new equilibrium state. Since the profile $< p > (\psi)$ is the same with and without the pellet, there is a tendency to return to the initial equilibrium state after the pellet cloud spreads out on magnetic field lines. However, when reconnection occurs, the profiles change and the pellet does not spread out to the initial equilibrium.

Simulations were also done with a simpler initialization in which the temperature was not modified by the $\rho_0/ < \rho >$ adiabatic correction. These simulations were quite similar in behavior, as far as the maximum displacement of the pellet cloud is concerned. However, since in this case the initial $p(\psi)$ profile was changed, the pellet had less tendency to return to the initial equilibrium.

## IV. Outboard Injection

All other things being equal, the 3D evolution depends on the initial location of the pellet perturbation in the poloidal plane, as well as the pellet density. In the case of outboard injection, the initial density $\rho$ is shown in Fig.4(a), in the plane $\phi = 0$ where the density is a maximum. The pellet is located at the outboard midplane. The density enhancement $\delta\rho/\rho_0$, where $\rho_0$ is the initial, uniform density, is 15. On a rapid time scale, during which there is approximate fast magnetosonic and shear Alfvén equilibration, the pellet expands outwards towards larger major radius.

Next, Fig.4(b) shows the density in the $\phi = 0$ plane at time $t = 96(R/a)\tau_A$, where $\tau_A$ is the Alfvén time defined in terms of the average toroidal magnetic field and background density. The pellet both shifts outwards and expands along the magnetic field.

The displacement of the pellet can be measured by calculating the flux surface average $< \rho >$, as in (8), using the toroidally averaged 2D magnetic field, its $n = 0$ component. This is acceptable if the initial and asymptotic states have axisymmetric magnetic field. In fact even at intermediate times, $\psi_0$, the $n = 0$ part of $\psi$, changes relatively little in comparison with the changes in the average density profile. The averaged density $< \rho > (\psi_0)$ is shown in Fig.5 at times (a) $t = 0$, (b) $t = 30$, and (c) $t = 96$. The maximum shift occurs at about $t = 30$. The peak of the $< \rho > (\psi_0)$ profile moves to lower $\psi_0$, which means that $< \rho >$ has moved closer to the wall boundary at $\psi = 0$. Afterwards, the peak starts to bounce back towards its initial state. In principle it should return to its initial state, if the density in each magnetic flux tube is conserved. However, there is some loss caused by density diffusivity $D$ and temperature diffusivity $\chi$, with $D = \chi = 5 \times 10^{-4} a^2 v_A/R$.

The three dimensional evolution of the density can be seen in Fig.6(a,b) and Fig.7(a,b), which are isoplots of the density surface at 80% of its maximum. Initially the density is localized near toroidal angle $\phi = 0$. In Fig.6(b)at time $t = 51$, the pellet has spread more than once around the torus, which gives an idea of the sound wave transit time for this case. In Fig.7(a), at time $t = 79$, the pellet material has spread out twice around the torus, and forms a "snake" like structure. The pellet material is not resonant with the $q = 2$ surface, since the pellet was injected on an initial $\psi$ surface with $q > 2$. Hence it smoothes out as time elapses, as seen in Fig.7(b) at $t = 96$.

## V. Inboard Injection and Reconnection

In the case of inboard injection, the initial density blob is located at the inboard midplane, shown in in Fig.8(a). The density enhancement $\delta\rho/\rho_0 = 10$. The density at $\phi = 0$, $t = 12.7(R/a)\tau_A$, is shown in Fig.8(b). The pellet has already reached the plasma center. In doing so, it elongates in shape in the radial direction. A further picture of the density at $\phi = 0$, $t = 65(R/a)\tau_A$, is shown in Fig.9. The average density profile is peaked at the plasma center, and the pellet material is nearly uniformly spread out on the flux surfaces.

The flux surface averages of $< \rho >$ are shown in Fig.10(a,b). The profile of $< \rho >$ has shifted the other way, to higher $\psi$, all the way to the center.

The three dimensional evolution of the density can be seen in Fig.11(a,b), which are isoplots of the density surface at 80% of its maximum. In Fig.11(a)at time $t = 12.7$, the pellet has spread once around the torus, similarly to Fig.6(b), in a sound wave transit time. In Fig.11(b), at time $t = 65$, the pellet material has spread out around the torus. The density forms a nearly axisymmetric structure peaked at the magnetic axis.

To get to this state, the density blob has to cut magnetic field lines, by the process of driven magnetic reconnection. Accordingly, the run was performed with a resistivity having minimum value $\eta = 2 \times 10^{-4} a^2 v_A / R$. The resistivity profile was proportional to the inverse equilibrium toroidal current, and the averaged Ohmic dissipation was balanced by a driving constant toroidal electric field. The effect of the pellet on the magnetic field can be seen in Fig.12, which shows contours of the magnetic flux $\psi$ in the $\zeta = 0$ plane at $t = 12.7$, when the pellet reaches the plasma center. It can be seen that the $\psi$ contours are pushed together opposite the pellet, at the location of the original $q = 2$ magnetic surface. The flux function $\psi$ is not constant on magnetic field lines in three dimensions, so it only gives an approximate picture of the magnetic field. However as seen from Fig.8(b) the density perturbation is roughly axisymmetric at $t = 12.7$, so Fig.12 represents approximately a two dimensional reconnection. The effect can also be seen in the averaged $q$ profile in Fig.13, in which the q profile is calculated with respect to the toroidal average of $\psi$. The q profile has reversed shear, because flux with $q > 2$ has replaced the original central flux having $q < 2$. This can be seen in comparision with Fig.3.

## VI. Intermediate Injection

In the case of intermediate injection, the initial density blob is located at the top, shown in in Fig.14(a). The density enhancement $\delta\rho/\rho_0 = 10$. The density at $\phi = 0$, $t = 150(R/a)\tau_A$, is shown in Fig.8(b). The pellet material is relatively uniformly spread around its initial magnetic surfaces.

The density peak, as it spreads along the magnetic field, also rotates, moving towards the outboard side. But by the time the peak reaches the outboard side, it is no longer big enough to cause outward displacement.

Incidentally, the motion of the peak around the magnetic surfaces is related to a mechanism of poloidal rotation spinup. To achieve a spinup, the density perturbation must be maintained against the smoothing effect of sound waves, either by a spatially dependent diffusion, or by injection of beams or pellets.

The flux surface averages of $<\rho>$ are shown in Fig.10(a,b). In intermediate cases, the pellet's motion is more tangent to the magnetic field, and there is relatively less motion of the averaged density peak. The initial averaged density profile is shown in Fig.10(a). At $t = 150$, the profile is given by Fig.10(b), with very little shift of the peak, although there is significant diffusive loss, with $D = \chi = 5 \times 10^{-4} a^2 v_A / R$.

## VII. Pellet Displacement

An approximate expression for the maximum pellet displacement can be derived as follows. In Reduced MHD, the equilibrium condition is

$$\mathbf{B} \cdot \nabla \nabla^2 \psi = -\frac{2}{R} \frac{\partial p}{\partial Z} \tag{14}$$

Approximate this equation by

$$-\frac{B}{qR}\left(\frac{a}{\delta a} + \frac{q}{\delta \phi}\right)\frac{\delta \psi}{(\delta a)^2} \approx -\frac{2}{R}\frac{\delta p}{\delta a}\cos\theta \tag{15}$$

where $\delta p$ is the pellet pressure, $\delta a$ is the pellet radius, and $\delta \phi$ measures the extent of the pellet cloud in the toroidal angle $\phi$. The factor $\cos\theta$, the component of the $R$ unit vector along the outward normal to the magnetic surface, accounts for the relative direction of the shift. The equilibrium $\psi$ at the pellet is approximately

$$\psi \approx -\frac{Ba^2}{qR}.$$

Combining these approximations, one finds that

$$\frac{\delta \psi}{\psi} \approx -\beta q^2 \frac{R}{a} \left(\frac{\delta a}{a}\right)^2 \left(1 + \frac{q \delta a}{a \delta \phi}\right)^{-1} \frac{\delta p}{p_0} \cos\theta \tag{16}$$

where $\beta = 2p_0/B^2$, and $p_0$ is the backgound pressure. Plugging in $\beta R/a = .30, \delta p/p_0 = 10, \delta a = .2a, q = 2.0$, and $\delta \phi = \pi/2$, gives

$$\frac{\delta \psi}{\psi} = -0.04 \frac{\delta p}{p_0} \cos\theta \tag{17}$$

The data from Fig.5, Fig.10 and Fig.15 is collected in Fig.16, which plots the maximum shift $\delta\psi/\psi$ as a function of $(\delta p/p_0)\cos\theta$. Included are other runs with $\delta\rho/\rho_0 = 2.5, 5.0$ with $\theta = 0$, as well as with $\delta\rho/\rho_0 = 7.5$ and $\theta = \pi$. Several cases were initialized with a simple non adiabatic model, in which the temperature was not modified to give an invariant pressure profile. The maximum displacement (16) is proportional to $\delta p$. In the adiabatic case $\delta p$ is given by (10), and in the nonadiabatic case by $\delta p = p_0\delta\rho/\rho_0$. The adiabatic case is marked by filled circles in Fig.16, and the simple case by open circles. The data is consistent with a straight line fit to the dotted line given by (17). The left most point from Fig.10, the inboard injection case, has the maximum possible deviation $\delta\psi/\psi$, the pellet having penetrated to the center.

It may be useful to express (16) in terms of a pellet displacement $\delta x = \delta\psi/\psi'$, the total pellet particle number $\delta N = \delta\rho(\delta a)^2 R\delta\phi$ and the background particle number $N_0 = \rho_0 2\pi^2 a^2 R$. Taking $\psi = 2\psi' a$ and $\delta p = p_0\delta\rho/ < \rho >$ gives

$$\frac{\delta x}{a} \approx \beta q^2 \frac{R}{a}(1 + \frac{q\delta a}{a\delta\phi})^{-1} \frac{\rho_0}{\rho_0 + < \delta\rho >} \frac{\delta N}{N} \frac{\pi^2}{\delta\phi} \cos\theta \tag{18}$$

The result (18) shows that sufficiently massive and localized pellets can have large excursions.

## VIII. Conclusion

Pellet injection in tokamaks has been simulated in an MHD model. The pellet is assumed to rapidly ablate and ionize, forming a dense plasma cloud. The cloud's formation is adiabatic, with no energy imparted to the plasma. The flux surface average of the pressure is the same, before and after pellet injection. The model advances the temperature so that it tends to be constant along the magnetic field. The density of the cloud is initially highly nonuniform on flux surfaces.

The simulations show that the plasma evolves through 3D states, ultimately tending towards a 2D equilibrium in which the density and temperature are flux functions. In the initial stages of this evolution, the cloud moves in the direction of increasing major radius. For pellets injected on the outboard side of the plasma, this means that the pellet cloud moves outwards, toward the wall enclosing the plasma. If a wall interaction model were included, part of the cloud's mass would have been lost to the wall. This is consistent with experiments, in which a significant fraction of the pellet mass is lost.

On the other hand, injection on the inboard side causes the cloud to move toward the magnetic axis. The pellet cloud is better confined then with outboard injection. This effect has also been seen in a recent experiment. We showed that if the pellet pressure perturbation is large enough, the effect is sufficient to carry the pellet all the way to the plasma center, an important result for fuelling or quenching of large tokamaks.

Moreover, magnetic reconnection driven by the pellet in this case gives rise to a negative central shear profile, which can be highly favorable for transport. Pellet injection might offer a way to maintain reversed central shear in large long pulse tokamaks.

Injection at the top (or bottom) causes relatively less pellet motion. This might be acceptable for experimental purposes, and would certainly be more accessible than inboard injection.

Finally, we showed that the numerical data is consistent with a simple model, in which the pellet displacement is proportional to the pressure enhancement of the pellet, and to the cosine of the angle between the outward normal to a flux surface at the pellet and the major radius direction. Within our parameter range, the data fit the model quite well. In the future, we plan to incorporate a more realistic ablation model, in higher resolution computations with smaller, denser, pellets, to see whether the model continues to hold. We also plan longer time simulations to track the approach to a two dimensional steady state.

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## References

[1] H. Strauss, and W. Park, "3D MHD Simulations on an Unstructured Mesh," Proceedings of 1997 International Sherwood Fusion Theory Conference, Madison, Wisconsin, paper 2C10, (1997)

[2] W. Park, Z. Chang, E. Fredrickson, G. Y. Fu, N. Pomphrey, H. R. Strauss, and L. E. Sugiyama, 3D Simulation Studies of Tokamak Plasmas using MHD and Extended - MHD Models, Sixteenth IAEA Fusion Energy Conference, Montreal, F1-CN-64/D2-2 (1996)

[3] D. Post, G. Janeschitz, A. Kukushkin, H. Pacher, M. Sugihara, J. Mandrekas, and W. Stacey, Phys. of Plasmas **4**, 2631 (1997).

[4] P. T. Lang, K. Buchl, M. Kaufmann, R. S. Lang, V. Mertens, H. W. Neuhauser, ASDEX Upgrade Team, NBI Team, Pellet injection into ASDEX Upgrade plasmas with improved scenario from the magnetic high - field side, Max - Planck - Institut fur Plasmaphysik, preprint, 1996.

[5] A. K. Macaulay, Geometrical, kinetic, and atomic physics effects in a 2D time dependent simulation of ablating fuel pellets, Nuclear Fusion **34**, 43 - 62 (1994).

[6] H. R. Strauss, Poloidal Rotation Spinup in Divertor Tokamaks, Physics of Plasmas *2*, 1229 - 1235, (1995).

[7] H. R. Strauss, Edge - localized mode simulations in divertor tokamaks, Phys. Plasmas *3*, 4095 (1996).

[8] W. Park, D. A. Monticello, H. Strauss, and J. Manickam, Physics of Fluids **29**, 1171 (1986).