# APMA E4306: Applied Stochastic Analysis

## Course Syllabus

**Instructor:** Shanyin Tong

**Email:** st3503@columbia.edu

**Lectures:** Mondays and Wednesdays 10:10-11:25 am, 524 Seeley W. Mudd Building

**Office hours:** Mondays and Wednesdays 11:30 am - 12:30 pm, 287 Engineering Terrace

**TA:** Daozhe Lin, dl3394@columbia.edu

**TA office hours:** Tue 10am-11am, Wed 4pm-5pm, 287 ET

**Course Description.** This course aims at providing an elementary introduction to fundamental ideas in stochastic analysis for applied mathematics. The course has four main parts: (i) a quick review of elementary probability theory (including limit theorems), and a short introduction to discrete Markov chains and Monte Carlo methods; (ii) elementary theory of stochastic process (e.g. continuous Markov process and Wiener process), Itô's stochastic calculus and stochastic differential equations; (iii) introductions to probabilistic representation of elliptic partial differential equations (the Fokker-Planck equation theory); and (iv) stochastic approximation algorithms and asymptotic analysis of SDEs. The main objective is not to give thorough technical studies on stochastic analysis, but rather to provide an elementary introduction to the fundamental concepts and tools in the topic and connect them to various applications in applied mathematics.

**Prerequisite.** Good understanding of probability theory (with law of large numbers and the central limit theorem) (on the level of IEOR E3658 or STAT G4001) and elementary stochastic process (on the level of the first part of IEOR E4106, STAT G4264 or STAT W5207 ) are required. Knowledge on elementary analysis (on the level of MATH GU4601), numerical methods (on the level of APMA E4300), and basic programming skills (with Python, MATLAB or equivalent) are also necessary.

**Website and Communication.** The chief means of communication for this course will be the course Canvas (aka Courseworks) site, accessed through https://courseworks2.columbia.edu/. Students are expected to check this for up-to-date assignments, including material separate from the text and announcements. Ed Discussions is an online discussion board for all issues related to content and logistics for the course. The communications and discussions will be conducted using this platform. You can access it through Canvas.

**Homework.** We employ an online grading system called Gradescope `https://gradescope.com/`. This should expedite the grading process and keep all assignments well-organized. You will be automatically enrolled, and the link can be accessed through Canvas. The assignments will be released and submitted through Gradescope.

Homework will be a mixture of theoretical questions and computational questions. You do not need to have experience coding beforehand, but you must be willing to learn basic programming during the class. A high-level software package that lets you easily plot things, such as Python or Matlab, is recommended.

**Academic Honesty.** You are encouraged to work with others, on the homework problems and to study. However, you must write up your own solutions. Students who violate university rules on academic dishonesty are subject to disciplinary penalties, including the possibility of failing the course and/or dismissal from the University. Detailed information on academic integrity at Columbia University is available here: `https://www.college.columbia.edu/academics/academicintegrity`.

**Grading.** 50 % Homework + 50% Final Exam

**Textbooks.** The course is based on Prof. Miranda Holmes-Cerfon's lecture notes: `https://cims.nyu.edu/~holmes/teaching.html#asanotes` and the following textbooks:

```
Applied Stochastic Analysis
Weinan E, Tiejun Li and Eric Vanden-Eijnden
American Mathematical Society, 2019

Probability and Random Processes
Geoffrey Grimmett and David Stirzaker
Oxford University Press, 2020

Stochastic Processes and Applications
Grigorios A. Pavliotis
Springer, 2015

Stochastic Differential Equations, Second Edition
Bernt Oksendal
Springer-Verlag, New York, 1995
```

Other good references:

```
Stochastic Approximation and Recursive Algorithms and Applications, Second Edition
Harold J. Kushner and G. George Yin
Springer-Verlag, Berlin, 1999

Stochastic Methods
Crispin Gardiner
```

Springer, 2009

Theory of Probability and Random Processes
Leonid Koralov and Yakov G. Sinai
Springer, 2007

Brownian Motion and Stochastic Calculus
Ioannis Karatzas, et al
Springer, 1991.

**Weekly Lecture Schedule.** Here is a rough schedule for the lectures.

| Week | Day | Topics |
|---|---|---|
| 1 | Jan 18 | Introduction & Review |
| 2 | Jan 23 & 25 | Discrete Markov chains |
| 3 | Jan 30 & Feb 1 | Detailed balance, Markov Chain Monte Carlo |
| 4 | Feb 6 & 8 | Continuous Markov chains |
| 5 | Feb 13 & 15 | Finite-dimensional distributions, Gaussian processes |
| 6 | Feb 20 & 22 | Stationary processes, Spectral theory |
| 7 | Feb 27 & Mar 1 | Brownian motion |
| 8 | Mar 6 & 8 | Stochastic Integration |
| - | Mar 13 & 15 | Spring Recess, No Classes Held |
| 9 | Mar 20 & 22 | Stochastic Differential Equations |
| 10 | Mar 27 & 29 | Numerically solving SDEs |
| 11 | Apr 3 & 5 | Forward and backward equations for SDEs |
| 12 | Apr 10 & 12 | Some applications of the backward equations |
| 13 | Apr 17 & 19 | Detailed balance, symmetry, and Eigenfunction expansions |
| 14 | Apr 24 & 26 | Stochastic gradient and related methods |
| 15 | May 1 | Asymptotic analysis of SDEs |
| | Schedule set by registrar | Final Exam |

This schedule is only tentative. Changes of the schedule will be announced in class. Homework will be handed out on the dates indicated.