

**APMA E2000: MULTIVARIABLE CALCULUS FOR ENGINEERS AND APPLIED  
SCIENTISTS  
SYLLABUS — FALL 2024**

**Instructors:**

- Shanyin Tong
- Drew C. Youngren

**Lectures:**

Section	Instructor	Email	Day/Time	Location
001	Shanyin Tong	<a href="mailto:st3503@columbia.edu">st3503@columbia.edu</a>	TR 8:40am–9:55am	Mudd 227
002	Drew Youngren	<a href="mailto:dcy2@columbia.edu">dcy2@columbia.edu</a>	TR 1:10pm–2:25pm	Chandler 402
003	Drew Youngren		TR 5:40pm–6:55pm	Hamilton 702

**Office Hours:**

Instructor	Day/Time	Location
Youngren	M 11:00am–1:00pm	Mudd 214
Youngren	R 10:00am–11:00am	Mudd 1106B
Tong	See CourseWorks	

**Recitations:** In addition to the lectures, you must also register for one of the recitation sections (E2001) listed in the [CU Bulletin](#).

Your TA will also host office hours established in the first session.

Section	Day/Time	Location	Instructor
R01	R 2:40pm–3:30pm	141 Uris Hall	Ling Lan
R02	R 4:10pm–5:00pm	602 Northwest Corner Building	Ling Lan
R03	F 10:10am–11:00am	414 Pupin Laboratories	Yinxi Pan
R04	R 11:40am–12:30pm	407 Mathematics Building	Yinxi Pan
R05	R 4:10pm–5:00pm	337 Seeley W. Mudd Building	Yinxi Pan
R06	F 2:40pm–3:30pm	414 Pupin Laboratories	Yin Zhou
R07	R 2:40pm–3:30pm	633 Seeley W. Mudd Building	Yin Zhou
R08	R 11:40am–12:30pm	337 Seeley W. Mudd Building	Yin Zhou
R09	F 1:10pm–2:00pm	307 Pupin Laboratories	Ling Lan

**Course description:** Multivariable Calculus is a third-semester calculus course for students who have previously been introduced to the basic ideas of differential and integral calculus in one variable.

In this course we will take the concepts of single-variable calculus (Calculus I and II) and look at their generalizations to functions of several (but often 2 or 3) variables. Topics include:

- Vectors and curves in space
- Functions of several variables, partial derivatives, gradients, and optimization
- Multiple integration, applications, and different coordinate systems
- Vector calculus including the theorems of Stokes, Gauss, and Green

The material we take up in this course has applications in physics, chemistry, biology, environmental science, astronomy, economics, statistics, and just about everything else. We want you to leave the course not only with computational ability, but with the ability to use these notions in their natural scientific contexts, and with an appreciation of their mathematical beauty and power.

**Textbook:**

- OpenStax, Calculus Volume 3. OpenStax CNX. Nov 8, 2018 <https://openstax.org/books/calculus-volume-3/>.
- Stewart, J. *Calculus: Early Transcendentals*. 8th Edition.

**Important Dates:** This page will be kept up-to-date with lecture topics and important events.

- Friday, 09/13, last day for change of program
- Thursday, 11/14, last day to drop or P/F
- Monday, 12/9, last day of classes
- Friday, 12/13, final exams begin (schedule pending)

**Policy on out-of-sequence exams and missed quizzes**

We are only able to accommodate a limited number of out-of-sequence exams. For this reason, we may approve out-of-sequence exams in the following cases:

- (1) A documented medical excuse.
- (2) A university-sponsored event such as an athletic tournament, a play, or a musical performance. Athletic practices and rehearsals do not fall into this category. Please have your coach, conductor, or other faculty advisor contact your instructor.
- (3) A religious holiday.
- (4) Extreme hardship such as a family emergency.

**We will not be able to accommodate out-of-sequence exams, quizzes, and finals for purposes of more convenient travel, including already purchased tickets. Please note again the date of the final and plan your summer break travel accordingly.**

Scheduled out-of-sequence exams and quizzes (those not arising from emergencies) must be taken before the actual exam. Makeups must occur within one week of the regularly scheduled exam or quiz, otherwise a zero score will be given.

**Grading:** The final grade will be computed with the following weights. For each category. Percentage will be converted to a letter according to the “college standard” chart at right. These cutoffs may be adjusted downward—they will never be made more strict—but not significantly.

Category	Weight	Grade	Minimum %
		A+	99
		A	93
Homework	25%	A-	90
		B+	87
Quizzes/Recitation	25%	B	83
		B-	80
Midterm Exams	30%	C+	75
		C	70
Final Exam	20%	C-	60
		D	50
		F	0

**A note on grades.**

Please refer the [SEAS Bulletin](#) for policy regarding the assignment of W, UW, and INC grades.

**Canvas:** The chief means of communication for this course will be the course Canvas (aka Courseworks) site, accessed through <https://courseworks2.columbia.edu/>. Students are expected to check this for up-to-date assignments—including material separate from the text—and announcements.

**Discussion Board:** Ed Discussions is an online discussion board for all issues related to content and logistics for the course.

**Homework:** Homework assignments come in two flavors:

**Practice:** Practice is essential to learning calculus. It is highly recommended each student keep a dedicated notebook for working practice problems. One should consult the relevant sections of the texts for such exercises, though suggested problems will often be posted.

**Written:** Weekly problem sets with a mix of exercises from the texts and supplementary problems will be collected. These include more in-depth problems requiring greater abstraction, understanding and/or synthesis of various concepts. In many ways, these constitute the heart of the course; rigor in their completion often yields the greatest understanding.

**Gradescope:** We employ an online grading system called Gradescope <https://gradescope.com/>. This should expedite the grading process and keep all assignments well-organized. You will be automatically enrolled, and the link can be accessed through Canvas.

**Recitation:** It is required that each student register for and attend a weekly recitation section. This is a venue for getting more individual help with content, to work more intricate problems, and receive richer feedback. Attendance will be taken. A participation grade will be added as an extra quiz grade at semester's end dependent on participation and effort. Your TA will give further details.

**Quizzes/Labs:** There will be a weekly, short assessment to check in on the fundamental concepts and/or an activity implementing them with a technological tool. Specific instructions and guidance will be laid out in recitation.

When calculating the homework and quiz grades for the semester, the lowest grades in each of these areas will be dropped. **N.B.** It is advised that students reserve these “passes” for unexpected absences. In fairness to all students and TAs, **late homework will not be accepted nor will make-ups be offered.**

**Technology:** Technology can play an important role in the learning of mathematics, and as such, graphing, scientific calculators, and computer algebra systems (CAS) are permitted for class and homework, though they will not be required. Calculators will not be permitted on tests and quizzes, and thus it is emphasized that students learn not to rely on them.

**Jupyter:** [Jupyter notebooks](#) are digital documents that combine formatted text and images with code that can be run in-place by a back-end server. Some materials will be provided via a [JupyterLite implementation](#). No programming skills are assumed or required, but students are encouraged to explore how the objects of calculus are represented and studied in a scientific computing environment.

**Academic Honesty:** Guidelines regarding cheating and plagiarism are laid out in the [SEAS Bulletin](#) and will be strictly adhered to. Collaboration is permitted, in fact encouraged, for home and class assignments; however, all submitted assignments must be written up independently and represent the student's own words and understanding.

**Schedule:** An **up-to-date course calendar** will be kept with references to relevant material.

Below is a proposed schedule for covering topics and assignments during the semester. All are subject to change as the semester progresses. Please refer to CourseWorks for all official dates.

Lec	Date	OpenStax	Stewart	Topic	Due	Note	Extras
1	9/3/2024	§2.1 2.2	§12.1–2	Coordinate systems, vectors			
2	9/5/2024	§2.3	§12.3–4	Dot Product/Cross product/Projection	HW0		<a href="#">Quadric Surfaces and Traces</a>
3	9/10/2024	§2.4 2.5	§12.5	Equations of lines and planes	HW1		
4	9/12/2024	§3.1–3	§13.1–3	Calculus of space curves		Last day for change of program	
5	9/17/2024	§3.4	§13.4	motion, velocity and acceleration	HW2		
6	9/19/2024	§3.3	§13.3	arc length, reparametrization, and curvature			
7	9/24/2024	§4.1 4.2	§14.1–2	Functions of several variables, continuity	HW3		
8	9/26/2024	§4.3	§14.3	Partial derivatives			<a href="#">Limit Examples</a>
9	10/1/2024	§4.4	§14.4	Tangent planes and linearization	HW4		
	10/3/2024			MIDTERM EXAM 1 (through 14.3)		lectures 1–8; hw 1–4	<a href="#">Exam 1 Sample</a>
10	10/8/2024	§4.5	§14.5	The chain rule	HW5		
11	10/10/2024	§4.6	§14.6	Directional derivatives and the gradient vector			
12	10/15/2024	§4.7	§14.7	Max/min values	HW6		<a href="#">Unconstrained Example</a>
13	10/17/2024	§4.8	§14.8	Lagrange multipliers			<a href="#">Lagrange Example</a>
14	10/22/2024	§5.	§15.1–4	Multiple/Iterated Integrals	HW7		<a href="#">Polar Integral</a>
15	10/24/2024	§5.4	§15.6–8	Polar/Cylindrical/Spherical Coordinates			
16	10/29/2024	§5.5	§15	Applications/Integration Review	HW8	Review for Midterm 2	
	10/31/2024			MIDTERM EXAM 2 (through 15.8)		lectures 9–16; hw 5–8	<a href="#">Exam 2 Sample</a>
	11/5/2024			University Holiday			
17	11/7/2024	§6.1	§16.1–2	Vector fields & Line Integrals			<a href="#">Potential Functions</a>
18	11/12/2024	§6.2	§16.3	Conservative fields	HW9		<a href="#">Velocity Fields</a>
19	11/14/2024	§6.3	§16.4	Work, Energy, Green's Theorem		Last day for drop or pass/fail	<a href="#">Work Integrals</a>
20	11/19/2024	§6.4–5	§16.6	Parametric surfaces/areas	HW10		
	11/21/2024			University Holiday			<a href="#">Green's and Div</a>
21	11/26/2024	§6.5–6	§16.5,7	Flux & Curl & Div	HW11		<a href="#">Winding Number</a>
22	11/28/2024	§6.7	§16.8	Stokes' Theorem			<a href="#">Parametric Surfaces</a>
23	12/3/2024	§6.8	§16.9	The Divergence Theorem	HW12		
24	12/5/2024			Final Review			
	12/6/2024			Review Session (reading week)			<a href="#">Old Final Exam</a>
	12/9/2024			FINAL EXAM		*projected dates, schedule set by registrar	