Assignment 6.

Due Oct 30.

1. We have a multinomial distribution with 4 possibilities $1, 2, 3, 4$ with probabilities $\{p_i(\theta)\}, i = 1, 2, 3, 4$ given by

$$p_1(\theta) = \frac{\theta}{3},\ p_2(\theta) = \frac{2\theta}{3},\ p_3(\theta) = \frac{2(1-\theta)}{3},\ p_4 = \frac{1-\theta}{3}$$

that depend on a parameter θ between 0 and 1. We have n independent observations $\{X_i\}$ each one being $1, 2, 3$ or 4. What is the MLE? What is $I(\theta)$ and the Cramér-Rao bound? Can you verify directly the asymptotic normality of the MLE with variance achieving the Cramér-Rao bound.? Use the fact that the frequencies $f_1, f_2, \ldots, f_k$ of an i.i.d sample from a multinomial with probabilities $p_1, p_2, \ldots, p_k$ satisfy a multivariate CLT with $E[f_i] = np_i$ and

$$E\Big[(\frac{f_i - np_i}{\sqrt{n}})^2\Big] = p_i(1 - p_i)$$

and for $i \neq j$

$$E\Big[(\frac{f_i - np_i}{\sqrt{n}})(\frac{f_j - np_j}{\sqrt{n}})\Big] = -p_i p_j$$

2. A sample of size 65 from a normal population with unknown mean μ and unknown variance θ had a sample mean of 2.1 with a sample variance of 0.64. Construct a 99% confidence interval for μ.