Probability, Limit Theorems

Problem set 4. Due Oct 17, 2002

Let $X_1, X_2, \ldots, X_n, \ldots$ be a sequence of independent random variables such that $E[X_i] = 0$ for $i = 1, 2, \ldots$. However they are not assumed to have the same distribution. We are interested in proving the weak law of large nukmbers, i.e. that

$$\lim_{n\to\infty} P\left[\left| \frac{X_1 + X_2 \cdots + X_n}{n} \right| \geq \epsilon \right] = 0$$

for every $\epsilon > 0$.

1. Show that the weak law of large numbers is not true in this generality by constructing a counterexample.

2. Show that it does not help even if we make the extra assumption that

$$\sup_n E[|X_n|] < \infty$$

3. Show that if we make the assumption that

$$\sup_n E[|X_n|^{1+\delta}] < \infty$$

for some $\delta > 0$, then the weak law holds.