Due Oct 9, 2000

Q1. Let $x(t)$ be a Gaussian process, with $E[X(t)] = 0$ and $\mathrm{Cov}[x(t)x(s)] = \rho(s,t)$. Let

$$\Delta(h) = \sup_{\substack{0\le s,t\le T\\ |t-s|\le h}} E[(x(t)-x(s))^2] = \sup_{\substack{0\le s,t\le T\\ |t-s|\le h}} [\rho(t,t)+\rho(s,s)-2\rho(s,t)].$$

If

$$\Delta(h) \le C[\log\frac{1}{h}]^{-\alpha}$$

for some C and $\alpha > 1$, show by the use of the GRR inequality that the Gaussian process can be constructed to have continuous paths.