Assignement 2. Due September 23, 2003

1. Let $\{x_n : n \geq 1\}$ be a sequence of points in $[0,1]$ and $\{p_n : n \geq 1\}$ a sequence of positive numbers such that

$$m = \sum_n p_n < \infty$$

Show that

$$\mu(A) = \sum_{n : x_n \in A} p_n$$

defines a set function $\mu(A)$ defined for all subsets $A \subset [0,1]$ which is a countably additive measure on $\mathcal{P}([0,1])$ the power set of $[0,1]$.

2. In the definition of Lebesgue measure, if we define

$$m[a,b] = F(b) - F(a)$$

(instead of $b - a$) for any interval $[a,b] \subset [0,1]$, where $F(x)$ is a continuous non decreasing function on $[0,1]$, show that the proof of the existence of a countably additive extension to the Borel sets $\mathcal{B}([0,1])$ can still be carried out.

3. What happens if $F(x)$ is nondecreasing but is allowed to have jumps?