Assignment 5. Due Oct 14, 2003

1. Let $f(x) \geq 0$ be a continuous function on $[0,1]$. We saw that if we define $f^*(x)$ by

$$f^*(x) = \sup_{|h| \leq 1} \frac{1}{2h} \int_{x-h}^{x+h} f(y)dy$$

then for some constant C, independent of f

$$\mu[x : f^*(x) \geq \ell] \leq \frac{C}{\ell} \int_0^1 f(x)dx$$

Show that a uniform estimate of the form

$$\int f^*(x)dx \leq C \int f(x)dx$$

cannot be valid for all non-negative continuous functions.

2. Let $f_n \geq 0$ and $f_n \to f$ in measure. Suppose

$$\lim_{n\to\infty} \int f_n(x)dx = \int f(x)dx$$

then show that

$$\lim_{n\to\infty} \int |f_n(x) - f(x)|dx = 0$$