**Assignment 7. Due Nov 4, 2003**

**Q 1.** Consider the following function $d(x,y)$ defined for irrational real numbers $E \subset R$.

$$d(x,y) = |x-y| + \sum_{j=1}^{\infty} \frac{1}{2^j} \frac{|\frac{1}{(x-r_j)} - \frac{1}{(y-r_j)})|}{1 + |\frac{1}{(x-r_j)} - \frac{1}{(y-r_j)}|}$$

where $\{r_j : j = 1, 2, \ldots\}$ is an enumeration of the rationals.

a) Show that $d$ is well defined on $E \times E$ and is a metric on $E$.

b) Show that if $x_n, x \in E$, $d(x_n, x) \to 0$ if and only if $|x_n - x| \to 0$.

c) Show that $(E, d)$ is a complete metric space.

d) Is $E$ with the usual metric $|x-y|$ complete?

**Q 2.** Abstractly the completion of a metric space $(X, d)$ is a complete metric space $(Y, D)$, a dense subset $Y_0 \subset Y$ and a one-to-one map $T : X \to Y_0$ such that $d(x_1, x_2) = D(Tx_1, Tx_2)$ for all $x_1, x_2 \in X$, i.e. an *isometry* between $(X, d)$ and $(Y_0, D)$. We proved in class that for any $(X, d)$ at least one completion exists. Show that the completion is unique. That is if $(Y_0, Y, D)$ and $(Y_0', Y', D')$ are both completions of $(X, d)$, then there exists a one to one onto map $U$ from $Y \to Y'$ with $D(y_1, y_2) = D'(Uy_1, Uy_2)$, i.e. an *isometry* between $(Y, D)$ and $(Y', D')$ such that $U$ maps $Y_0$ onto $Y_0'$.