Real Variables Fall 2007.

Assignment 11. Due Nov 19.

Problem 1. Let $f(x)$ be a continuous function of x defined on $0 \leq x \leq 1$. Show that the following sequence $p_n(x)$ of polynomials of degree n converges to $f(x)$ uniformly on $[0,1]$ as $n \to \infty$.

$$p_n(x) = \sum_{j=1}^{n} \binom{n}{j} f(\frac{j}{n}) x^j (1-x)^{n-j}$$

HINT: Use the identities

$$\sum_{j=1}^{n} \binom{n}{j} j\, x^j (1-x)^{n-j} = n\, x$$

and

$$\sum_{j=1}^{n} \binom{n}{j} (j - nx)^2\, x^j (1-x)^{n-j} = n\, x\, (1-x)$$

Problem 2. X is a compact metric space. $C(X)$ is the space of (bounded) continuous functions with $d(f,g) = \sup_x |f(x) - g(x)|$. A real valued function $f : X \to R$ is Lipschitz continuos on X if there exists a $C < \infty$ such that for any $x, y \in X$,

$$|f(x) - f(y)| \leq C\, d(x,y)$$

For any λ, show that the function $g_\lambda(x) = \sup_y [f(y) - \lambda\, d(x,y)]$ is Lipschitz continuous and that

$$\sup_x |g_\lambda(x) - f(x)| \to 0 \quad \text{as} \quad \lambda \to \infty$$