

**Real Variables Fall 2007.**

**Assignment 3. Due Sept 24.**

**1.** Let  $\mu$  be a countably additive measure on the Borel  $\sigma$ -field  $\mathcal{B}(R)$  of the real line  $R$ , with  $\mu(R) = 1$ . For  $-\infty < x < \infty$  define

$$F(x) = \mu[(-\infty, x]] = \mu[\{y : -\infty < y \leq x\}]$$

Show that  $F(x)$  satisfies the following properties:

(i).  $F(x)$  is nondecreasing.

(ii).  $F(x)$  is right-continuous. i.e.  $F(x+0) = F(x)$  for every  $x \in R$ .

(iii).

$$\lim_{x \rightarrow -\infty} F(x) = 0; \quad \lim_{x \rightarrow \infty} F(x) = 1$$

**2.** Conversely if  $F(x)$  on  $R$  satisfies (i)-(iii) above show that there is a unique  $\mu$  on  $\mathcal{B}(R)$  such that

$$F(x) = \mu[(-\infty, x]]$$