

Real Variables Fall 2007.

Assignment 4. Due Oct 1.

1. Let $\{f_n(x) : n \geq 1\}$ be a sequence of measurable functions on (X, \mathcal{B}) . Show the following:

(i).

$$E = \{x : \sup_n f_n(x) < \infty\}$$

is measurable.

(ii)

$$A = \{x : \lim_{n \rightarrow \infty} f_n(x) = f(x) \text{ exists} \}$$

is measurable.

(iii). The limiting function $f(x)$ in (ii) is measurable on A .

2. Show that if $f_n(x) \rightarrow f(x)$ for almost all x , then for any $\epsilon > 0$,

$$\lim_{n \rightarrow \infty} \mu[x : |f_n(x) - f(x)| \geq \epsilon] = 0$$