Real Variables Fall 2007.

Assignment 5. Due Oct 10.

1. Integration with respect to σ-finite measures. Let A_n be disjoint measurable sets with $X = \cup A_n$ and $\mu(A_n) < \infty$. One can define integrability of a non-negative measurable f on X in two ways.

(i). f is integrable if

$$\int f \, d\mu = \sum_n \int_{A_n} f \, d\mu < \infty$$

or

(ii). f is integrable if it is integrable on any set A of finite measure and

$$\int f \, d\mu = \sup_{\substack{A \in \mathcal{B} \\ \mu(A) < \infty}} \int_A f \, d\mu$$

Show that both definitions are equivalent and define the same value for the integral.

2. Prove Fatou's lemma for σ-finite measures.