Problemset 10

Q1. $\mathcal{X}$ and $\mathcal{Y}$ are Banach spaces. T_n is a sequence of bounded operators $\mathcal{X} \to \mathcal{Y}$ such that $\sup_n \|T_n\| \leq C$ and

$$\lim_{n\to\infty} T_n x = Tx$$

exists for $x \in D$, a dense subspace of $\mathcal{X}$. Show that

$$\lim_{n\to\infty} T_n x = Tx$$

exists for all $x \in \mathcal{X}$ and $\|T\| \leq C$.

Q2. If $\mathcal{X}$ and $\mathcal{Y}$ are Hilbert spaces and T is an isometry between dense subspaces $D_1 \subset \mathcal{X}$ and $D_2 \subset \mathcal{Y}$, then T extends as an isometry between $\mathcal{X}$ and $\mathcal{Y}$.

Q3. The space $\mathcal{S}$ of functions on R consists of smooth functions that satisfy for nonnegative integers n and r

$$|\frac{d^n f(x)}{dx^n}| \leq C_{r,n}(1+x^2)^{-r}$$

for some constants $C_{r,n}$. Show that $f \in \mathcal{S}$ if and only if it Fourier transform

$$(\widehat{f})(x) = \int e^{ixy} f(y) dy \in \mathcal{S}$$