1. F satisfies a Lipschitz condition if $|F(x)-F(y)| \leq C|x-y|$ for some constant C. Show that such a function is differentiable almost everywhere with respect to Lebesgue measure and

$$F(b)-F(a)=\int_a^b F'(x)dx.$$

2. Construct a function $F(x)$ on $[0,1]$ that is nondecreasing $F(0)=0, F(1)=1$ and $F'(x)=0$ a.e. with respect to Lebesgue measure. Hint: Start with $f(x)=x$. Divide $[0,1]$ into three equal parts. Change it to $f(x)=\frac{3}{2}x$ on $[0,\frac{1}{3}]$, equal to $\frac{1}{2}$ on $[\frac{1}{3},\frac{2}{3}]$ and $\frac{1}{2}+\frac{3}{2}(x-\frac{2}{3})$ on $[\frac{2}{3},1]$ [Helps to draw a picture]. Repeat indefinitely leaving the middle piece constant. After n steps you will have n flat pieces and 2^n pieces of slope $(\frac{3}{2})^n$ on intervals of size $(\frac{1}{3})^n$. The limit will exist and should work.

3. Take a number from $[0,1]$. Expand it in binary expansion as $x=\sum_{j=1}^{\infty}\frac{a_j}{2^j}$ where $a_j=0,1$. Make it unique by replacing eventual $1'$s by $0'$s. Define the map $T(x)=\sum_{j=1}^{\infty}\frac{b_j}{3^j}$ where $b_j=0$ if $a_j=0$ and $b_j=2$ if $a_j=1$. If μ is Lebesgue measure then μT^{-1} will provide a counter example. Any connection to problem **2**?