Problemset 9.

1. Find the eigenvalues and eigenvectors of the operator on $L_2[0,1]$ with Lebesgue measure given by

$$(Tf)(s) = \int_0^1 K(s,t)f(t)dt$$

where $K(s,t) = \min\{s,t\} - st$.

2. If A and B are two self adjoint compact operators on a Hilbert space $\mathcal{H}$ that commute, i.e. $AB = BA$, show that there is a common orthonormal set of eigenvectors $\{e_j\}$ such that $Ae_j = a_j e_j$ and $Be_j = b_j e_j$ with real constants a_j, b_j

3. What would the result look like if you dropped the assumption of compactness but the operators A, B commute and are bounded and self adjoint.