$X(t)$ is the gamma process with Levy-Khintchine representation

$$E[e^{itX(1)}] = \exp[\int_0^\infty [e^{itx} - 1]\frac{e^{-x}}{x}dx$$

Let $\ell > 0$, Write the Levy measure as the sum

$$\frac{e^{-x}}{x}dx = \frac{e^{-x}}{x}\mathbf{1}_{x\le\ell}dx + \frac{e^{-x}}{x}\mathbf{1}_{x\ge\ell}dx$$

with tow processes $X_\ell(t)$ and X^ℓ consisting entirely of jumps that are smaller than ℓ and larger than ℓ respectively. If Y is the largest jump of $X(t) = X_\ell(t)+X^\ell(t)$ during $0 \le t \le 1$, then the joint distribution of $X = X(1)$ and Y can be computed.

$$\begin{aligned}
P[X \le b, Y \le \ell] &= P[X^\ell(1) = 0, X^\ell(1) \le b] \\
&= P[X^\ell(1) = 0] \times P[X^\ell(1) \le b] \\
&\exp[-\int_\ell^\infty \frac{e^{-x}}{x}dx] \times \int_0^b f_\ell(y)dy
\end{aligned}$$

where

$$f_\ell(y) = \frac{1}{2\pi}\int_{-\infty}^\infty \exp[\int_0^\ell [e^{itx} - 1]\frac{e^{-x}}{x}dx]e^{-ity}dt$$

Change the contour to $-i - \infty, -i + \infty$, i.e $t \to -i + t$. Then

$$\begin{aligned}
f_\ell(y) &= \frac{1}{2\pi}\int_{-\infty}^\infty \exp[\int_0^\ell [e^{x+itx} - 1]\frac{e^{-x}}{x}dx]e^{-y-ity}dt \\
&= \frac{e^{-y}}{2\pi}\int_{-\infty}^\infty \exp[\int_0^\ell [e^{itx} - e^{-x}]\frac{dx}{x}]e^{-ity}dt \\
&= \frac{e^{-y}}{2\pi}\exp[\int_0^\ell \frac{1-e^{-x}}{x}dx]\int_{-\infty}^\infty \exp[\int_0^\ell [e^{itx} - 1]\frac{dx}{x}]e^{-ity}dt \\
&= \frac{e^{-y}}{2\pi}\exp[\int_0^\ell \frac{1-e^{-x}}{x}dx]\int_{-\infty}^\infty \exp[\int_0^1 [e^{it\ell x} - 1]\frac{dx}{x}]e^{-ity}dt \\
&= \frac{e^{-y}}{2\pi\ell}\exp[\int_0^\ell \frac{1-e^{-x}}{x}dx]\int_{-\infty}^\infty \exp[\int_0^1 [e^{itx} - 1]\frac{dx}{x}]e^{-it\ell^{-1}y}dt \\
&= \frac{e^{-y}}{\ell}\exp[\int_0^\ell \frac{1-e^{-x}}{x}dx]\phi(\frac{y}{\ell})
\end{aligned}$$

with

$$\phi(y) = \frac{1}{2\pi}\int_{-\infty}^\infty \exp[\int_0^1 [e^{itx} - 1]\frac{dx}{x}]e^{-ity}dt$$