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Q 1. Let  $x_1, \dots, x_n$  be  $n$  independent observations from a normal population with both mean and variance equal to an unknown parameter  $\theta$ . Is there a sufficient statistic? What is the MLE? What is the Cramér-Rao lower bound? Is it reached asymptotically? What is the asymptotic efficiency of the mean relative to the MLE?

Q 2. Draw a random sample of size 25 from a Cauchy distribution symmetric about 0. Calculate the median and the first approximation to the MLE of the location parameter  $\theta$  in  $\frac{1}{\pi} \frac{1}{1+(x-\theta)^2}$ . Repeat the experiment 25 times. Do you see the improvement in the efficiency of the approximate MLE over the median? Is it by the expected amount?