Assignment 2.

1. For $p > \frac{1}{2}$ consider the Markov Chain on $Z^+ = \{0, 1, \ldots, n, \cdots\}$ defined by the transition probabilities

$$\pi(x, x+1) = p$$
$$\pi(x, x-1) = 1 - p$$

for $x \geq 1$ and $\pi(0,1) = 1$. The remaining $\pi(\cdot, \cdot)$ are 0. Show that the chain $\xi_n \to \infty$ as $n \to \infty$. What is the probability that if it starts from $x > 0$ it never visits 0?

2. For the same Markov Chain if $p < \frac{1}{2}$, show that the process visits 0 with probability 1 and calculate

$$m(x) = E[\tau | \xi_0 = x]$$

where $\tau = \{\inf n : \xi_n = 0\}$ is the time of the first visit to 0.