Assignment 5.

1. Another place to start is smooth functions f for which one can define (of course $x(0) = 0$)

$$I(f) = \int_0^T f(s)dx(s) = f(T)x(T) - x(0)f(0) - \int_0^T x(s)f'(s)ds$$

Show that if $f_n \to f$ in $L_2[0,T]$ then again the limit $I(f_n) = I(f)$ exists in $L_2(P)$ and defines a Gaussian random variable with mean 0 and variance equal to $\int_0^T |f(t)|^2 dt$.

2. Take $T = 1$, the partition $t_j = \frac{j}{2^n}$ and define

$$V_n = \sum_{j=0}^{2^n - 1} |x(\frac{j+1}{2^n}) - x(\frac{j}{2^n})|$$

Compute $E[V_n]$ and $Var[V_n]$. Show that $E[V_n] \to \infty$ and $Var[V_n]$ remains bounded. Since $V_n \uparrow$ in n, $V_\infty = \lim_n V_n$ exists. Now use the information you have to show that $V_\infty = +\infty$ with probaility 1.