Assignment 7.

1. $\beta(t)$ is Brownian motion. $x(t) = \arctan \beta(t)$. Is $x(t)$ a Markov process? Does it satisfy a stochastic differential equation with respect to the Brownian motion $\beta(t)$. Does it satisfy

$$dx(t) = \sigma(x(t))d\beta(t) + b(x(t))dt$$

for some σ and b. If so write it down.

2. Itô's formula for Brownian Motion says

$$f(\beta(t)) = f(\beta(0)) + \int_0^t f'(\beta(s))d\beta(s) + \frac{1}{2}\int_0^t f''(\beta(s))ds$$

provided f is twice continuously differentiable. Apply it to the function $f(x) = |x|$ in order to define

$$A(t,\omega) = \int_0^t \delta(\beta(s))ds$$

Show that

$$A(t,\omega) = \lim_{\epsilon \to 0} \frac{1}{2\epsilon} \int_0^t \chi_{[-\epsilon,\epsilon]}(\beta(s))ds$$

exists.