Basic Algorithms – Sample Final Exam

(1) Express the total time $T(n)$ required for this function as a sum. What is the value returned by this function?

```
FUNCTION myst(n)
    r := 0;
    FOR i:= 1 TO n-1 DO
       FOR j := i+1 TO n DO
          FOR K := 1 TO J DO
             r := r+1
    RETURN(r)
```

(2) Suppose that a heap is stored in an array H in standard fashion. That is, the root is in $H[1]$, and for every node stored in $H[k]$, store its children in $H[2k]$ and $H[2k+1]$. Prove that an n-node heap occupies the first n contiguous entries of H.

(3) Solve the following recurrence relations exactly.

$$a)\ \ T(n) = T(n-2) + 3n + 4 \text{ for } n \geq 3, \quad T(1) = 1, T(2) = 6.$$

$$b)\ \ T(n) = 2T(n/2) + 6n - 1, \quad T(1) = 1.$$

(4) Construct a heap containing the items 10,2,9,16,8,6,1,3,12 using the BUILDHEAP algorithm. Draw the heap at each step.

(5) Write an algorithm (pseudo-code) for calculating the number of descendants of each vertex in a tree.

(6) Suppose you are given a sorted list of N elements followed by $f(N)$ randomly ordered elements. How would you sort the entire list if

a. $f(N) = O(1)$? *b.* $f(N) = O(\log N)$? *c.* $f(N) = O(\sqrt{N})$.

(7) Show the contents of the *known*/*d*/*p* table after each iteration of Dijkstra's algorithm executed on the graph below with source vertex 1.

(8) Draw the spanning forest after every iteration of Kruskal's algorithm on the preceding graph (igonoring the directions on the edges).

(9) The travelling salesman problem is defined as follows. Given a directed graph with weights, find a Hamiltonian cycle of minimal total cost. A simple "greedy algorithm" for this starts at node 1 and moves along the edge of minimum cost to the next node, and repeats this until it finds a node that it has visited before, or can go no further. Show that this may fail.

(10) An undirected graph is said to be 2-colorable if all the vertices can be colored such that no two adjacent vertices have the same color. Describe an algorithm that runs in time $O(|V| + |E|)$ to color a graph with two colors or determine that the graph is not 2-colorable.

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$