New York University
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Computer Exercises

Objective: To explore different versions of the QR factorization and its application.

1. Use the standard Householder QR to solve the problem 3.2, Page 153. In Matlab, the function call for Householder QR is `[q,r]=qr(A)`. The so-called best values of the altitudes are simply the least square solution to the linear system. Omit the questions given in problem 3.2, instead, answer the following questions
(i) Provide the matrix on which you apply QR factorization; is it tall or flat
(ii) Print out the solution $x(1:4)$ in short e format, as in `3.1416e+00` for π
(iii) What is the quantity your solution minimizes.

2. Given m, n as input, write two Matlab *functions* to construct the Hilbert matrix $H \in \mathbb{R}^{m\times n}$ defined by $H_{ij} = 1/(i+j-1)$. The first version `hilb1(m,n)` uses two embedded *for-loops*. The second version `hilb2(m,n)` is fully vectorized (Hint: Consider the matrix `B=transpose(1:m)*ones(1,n)`, and what does `1./B` do). Answer the following questions
(i) Print out the matrix H in short e format for $m = 8$, $n = 5$
(ii) Provide the Matlab script for your vectorized version
(iii) Write and provide a script of no more than 12 lines to call the functions for the case $m = 2000$, $n = 1600$. Use `cputime` to show the runtimes for the two functions.
(iv) Show the condition number of H for case $m = 8$, $n = 5$ and the case $m = 32$, $n = 16$.

3. Given $A \in \mathbb{R}^{m\times n}$ as input, write a Matlab *function* `[Q,R]=qrmgs(A)` to implement the modified Gram-Schmidt QR; see Page 132. You must have the two inner for-loops vectorized, and please keep this code for our future project
(i) Provide Matlab script for the function, with two inner for-loops vectorized
(ii) Run it on the Hilbert matrix H with $m = 8$, $n = 5$. Do not provide the result Q or R, instead, calculate and show the two backward errors

$$\|Q^T Q - I\|_2 = \texttt{norm(transpose(Q)*Q - eye(n))}, \quad (1)$$

$$\frac{\|Q R - H\|_2}{\|H\|_2} = \texttt{norm(Q*R - H) / norm(H)} \quad (2)$$

(iii) Repeat (ii) for $m = 32$, $n = 16$
(iv) Repeat (ii) and (iii) using the Householder QR