Assignment 6

Write a (Matlab) code to implement the fast Poisson solver. Note: the standard forward and backward Fast Fourier transforms, as implemented in Matlab or elsewhere, are not unitary transforms. If we denote by F and B as the forward and backward transforms, $F \cdot B = B \cdot F = n \cdot I$; in fact, $1/\sqrt{n} \cdot F$ and $1/\sqrt{n} \cdot B$ are two unitary transforms, one being inverse to the other.

Use your code combined with the fourth order finite difference scheme to solve the equation

$$\nabla^2 u(x,y) = f(x,y), \quad (x,y) \in D = [0,\pi] \times [0,\pi] \tag{1}$$

subject to the Dirichlet boundary conditions $u(x,y)|_{\partial D} = g(x,y)$.

(a) Check the correctness of your code by choosing special f and g for which you know the exact solution. You need to invent your own way to convince me and youself that the code you write has no bugs.

(b) Once you debug the code, use sufficient number of points to solve the following problem for which $f = 0$, and g is such that

$$\begin{aligned} u(0,y) &= \sin(ky), \quad y \in [0,\pi], & (2)\\ u(\pi,y) &= \sin(ny), \quad y \in [0,\pi], & (3)\\ u(x,0) &= 0, \quad x \in [0,\pi], & (4)\\ u(x,\pi) &= 0, \quad x \in [0,\pi]. & (5) \end{aligned}$$

Make a surface plot for $z = u_h(x,y)$ for $k = 8$, $n = 16$.

(c) Optional. Explain the behavior of the solution.