Assignment 6

1. Use the matlab function ode45 (provided in Matlab) to solve the Lorenz equations specified on page 229 of Lindfield's text (a syetem of three equations), and make the two plots as given on page 230: one is a plot in the so-called phase space, the x-z space; the other is for the three solutions as functions of t

2. 3. 4. Problem 5.4, 5.11, 5.14

5. For $k = 30, \mu = 0.80$, solve the initial value problem (the Riccati equation)

$$\begin{cases} y' = i \cdot k\{y^2 - [1 + \mu \cdot \sin(5t)]\}, & 0 \le t \le 2\pi, \\ y(0) = 1, \end{cases} \tag{1}$$

by implementing the following two schemes on a equispaced grid with $h = 2\pi/N$, $t_n = n \cdot h$.

(i) The fourth order, four-stage, classical ERK method (5.5.1), page 211

(ii) The implicit scheme

$$y_{n+1} = y_n + i \cdot hk\{y_n \cdot y_{n+1} - [1 + \mu \cdot \sin(5t_{n+1/2})]\} \tag{2}$$

Hint: See page 2 of this assignment.

a. Check their rates of convergence numerically.

b. For each case, find the least number of points N, or its reasonable approximation, such that your solution at $t = 2\pi$ has relative error no less than (i) 10^{-2}, and 10^{-4}, for the classical ERK (ii) 10^{-2} for the scheme (2)

c. Plot the numerical solutions as functions of $t \in [0, 2\pi]$.

d. Extra. Use the Richardson's extrapolation to improve the accuracy of the numerical solution at $t = 2\pi$ obtained from the scheme (2). Namely, use $N_1 = N$, $N_2 = \lfloor 1.3N \rfloor$ for the two runs of the scheme (2) in order to find $A(0)$ as defined by the formula

$$A(0) = A(h) + c \cdot h^\alpha + O(h^{\alpha+2}), \tag{3}$$

where $A(0)$ is the exact solution at $t = 2\pi$, $A(h)$ is the numerical solution at $t = 2\pi$ and with a specific h value.

Hint: First proceed with 1.3 replaced by 2.

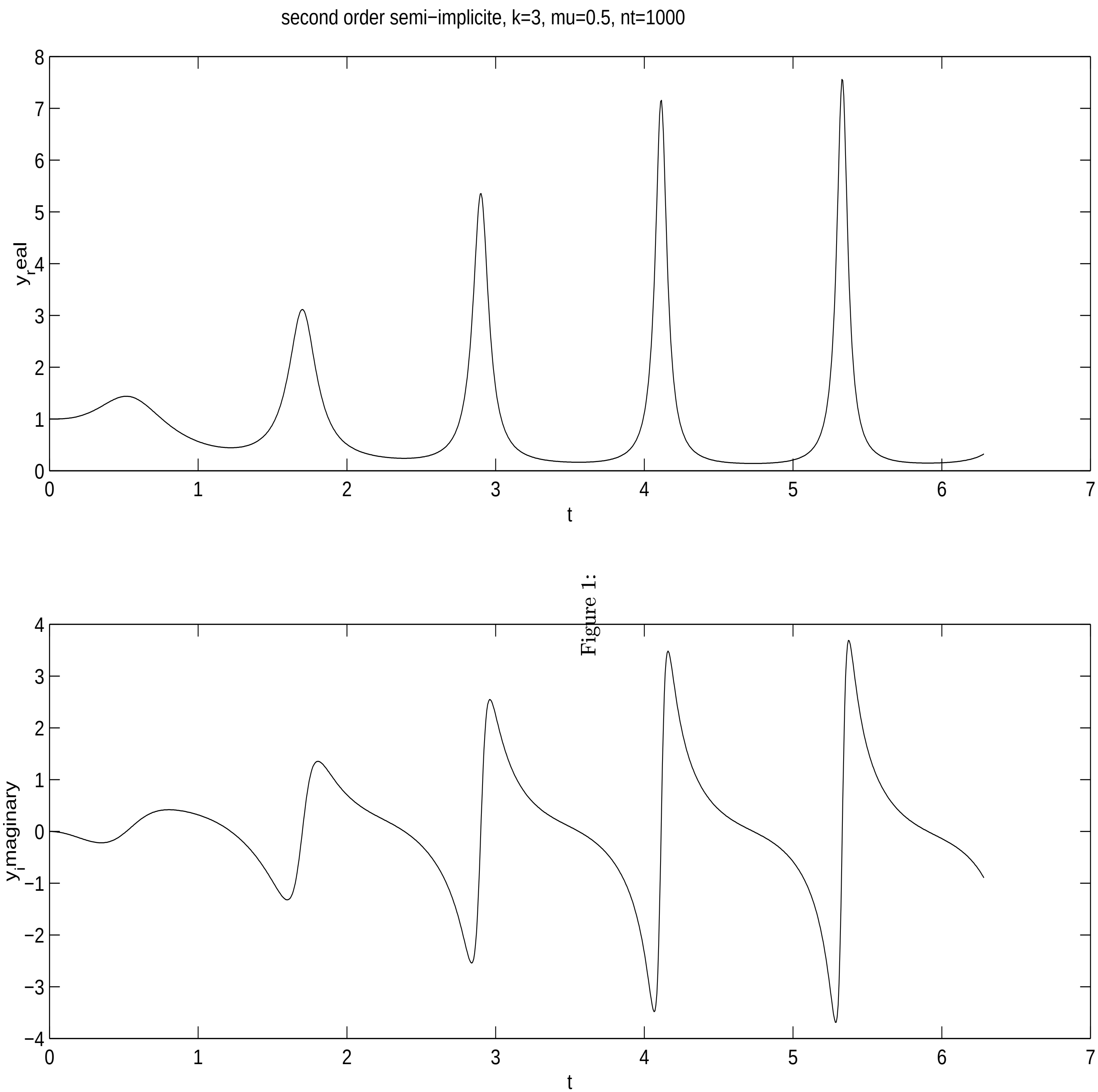
second order semi−implicite, k=3, mu=0.5, nt=1000
y_real
t
y_imaginary
t

Figure 1: