MODULARITY OF GALOIS REPRESENTATIONS

We will give a leisurely introduction to the connection between modular forms and Galois representations as it evolved through the course of the last century. Some of its inspiration came from the remarkable congruences Ramanujan found for his function $\tau(n)$. These were explained by Serre and Swinnerton-Dyer using Galois representations attached to Ramanujan's $\Delta$-function. These two functions are defined by

$$\Delta(e^{2\pi i z}) = e^{2\pi i z} \prod_n (1 - e^{2\pi i n z})^{24}.$$ 

We will build up to the statement of Serre's conjecture in this context, and describe its solution in joint work with J-P. Wintenberger.