That the linear systems arising upon the discretization of elliptic PDEs can be solved very efficiently is well-known, and many successful iterative solvers with linear complexity have been constructed (multigrid, Krylov methods, etc). Interestingly, it has recently been demonstrated that it is often possible to directly compute an approximate inverse to the coefficient matrix in linear or close to linear) time. The talk will survey some recent work in the field and will argue that direct solvers have several advantages, including improved stability and robustness, and dramatic improvements in speed in certain environments. Moreover, the direct solvers being proposed have low communication costs, and appear to be better suited to parallel implementations than many currently existing fast solvers.